## STAT 3743: Probability and Statistics

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Fall 2010
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## Quantitative Data

Notes

Quantitative data: any that measure the quantity of something

- invariably assume numerical values $\qquad$
- can be further subdivided:
- Discrete data take values in a finite or countably infinite $\qquad$ set of numbers $\qquad$
- Continuous data take values in an interval of numbers. AKA scale, interval, measurement
- distinction between discrete and continuous data not
$\qquad$
always clear-cut $\qquad$
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## Example

Annual Precipitation in US Cities. (precip) avg amount
Notes
rainfall (in.) for 70 cities in US and Puerto Rico.

```
> str(precip)
> precip[1:4]
Mobile Juneau Phoenix
\begin{tabular}{lll}
67.0 & 54.7 & 7.0
\end{tabular}
Little Rock 48.5
```

Named num [1:70] 6754.7748 .51417 .220 .71343 .440 .2 ...

- attr(*, "names")= chr [1:70] "Mobile" "Juneau" "Phoenix" "Little Rock" ...


## Example

Lengths of Major North American Rivers. (rivers)
lengths (mi) of rivers in North America. See ?rivers.

```
> str(rivers)
```

num [1:141] $735320325392524 \ldots$
> rivers[1:4]
[1] 735320325392

```
Example
Yearly Numbers of Important Discoveries.
(discoveries) numbers of "great" inventions/discoveries in
each year from 1860 to 1959 (from 1975 World Almanac)
> str(discoveries)
Time-Series [1:100] from 1860 to 1959: 5 3 0 2 0 3 2 3 6 1 ...
> discoveries[1:4]
[1] 5 3 0 2
```

- Strip charts (or Dot plots):
- for either discrete or continuous data
- usually best when data not too large. $\qquad$
the stripchart function
three methods $\qquad$
- overplot - only distinct values $\qquad$
- jitter - add noise in $y$ direction $\qquad$
- stack - repeats on top of one another
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## Displaying Quantitative Data

- Strip charts (or Dot plots):
- for either discrete or continuous data
- usually best when data not too large.
- the stripchart function
- three methods:
- overplot - only distinct values
- jitter - add noise in $y$ direction
- stack - repeats on top of one another

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```
> stripchart(precip, xlab = "rainfall")
> stripchart(rivers, method = "jitter",
+ xlab = "length")
> stripchart(discoveries, method = "stack",
+ xlab = "number")
```

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Notes

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Figure: Stripchart of precip
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Figure: Stripchart of rivers
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Figure: Stripchart of discoveries

- Histograms
- typically for continuous data
- decide on bins/classes, make bars proportional to membership
- often misidentified (bar graphs)
> hist (precip, main = "")
> hist (precip, freq = FALSE, main = "") $\qquad$
$\qquad$
$\qquad$
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Figure: Histograms of precip $\qquad$

- choose different bins, get a different histogram
- many algorithms for choosing bins automatically
- should investigate several bin choices
- look for stability
- try to capture underlying story of data $\qquad$
$\qquad$
$\qquad$
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Stemplots

- Stemplots have two basic parts: stems and leaves
- initial digit(s) taken for stem
- trailing digits stand for leaves
- leaves accumulate to the right

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## Example

Road Casualties in Great Britain 1969-84. A time series of total car drivers killed or seriously injured in Great Britain monthly from Jan 1969 to Dec 1984.

```
> library(aplpack)
> stem.leaf(UKDriverDeaths, depth = FALSE)
1 | 2: represents 120
    leaf unit: 10
        n: }19
        10 | 57
        l11:136678
    12 123889
    |}00001222344
    ,000122234444455555666778888
    | 0000111112222223444455555566677779
    16 | 01222333444445555555678888889
    17 | 11233344566667799
    17 18 00011235568
    18|00011235568
    \0 0000113557788899
    20 | 0000113557788899
    21|145599
    22 | 013467
    23|9
    2319
HI: 2654
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```


## Code for stemplots

> UKDriverDeaths[1:4]
[1] 1687150815071385
> stem.leaf(UKDriverDeaths, depth = FALSE) $\qquad$
1 | 2: represents 120
leaf unit: 10

$$
\mathrm{n}: 192
$$

10 | 57
11 | 136678
12 | 123889
13 | 0255666888899
14 | 00001222344444555556667788889
15 | 0000111112222223444455555566677779
 17 | 11233344.566667799

Good for plotting data ordered in time

- a 2-D plot, with index (observation number) on $x$-axis,
$\qquad$
value on $y$-axis
- two methods
- spikes: draws vertical line up to value (type $=$ " h ")
- points: simple dot at the observed height (type $=$ " p ") $\qquad$
Example
Level of Lake Huron 1875-1972. annual measurements of $\qquad$
the level (in feet) of Lake Huron from 1875-1972. $\qquad$
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## Index Plots

Good for plotting data ordered in time

- a 2-D plot, with index (observation number) on $x$-axis, value on $y$-axis
- two methods
- spikes: draws vertical line up to value (type = "h")
- points: simple dot at the observed height (type = "p")


## Example

Level of Lake Huron 1875-1972. annual measurements of
Notes
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$\qquad$ the level (in feet) of Lake Huron from 1875-1972. $\qquad$


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Figure: Index plots of LakeHuron
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## Qualitative Data, Categorical Data, Factors

- Qualitative data: any data that are not numerical, or do

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- Factors subdivide data into categories
- possible values of a factor: levels
- factors may be nominal or ordinal $\qquad$
- nominal: levels are names, only (gender, political party, $\qquad$ ethnicity)
- ordinal: levels are ordered (SES, class rank, shoe size) $\qquad$


## Example

U.S. State Facts and Features. postal abbreviations

```
> str(state.abb)
```

$\qquad$
chr [1:50] "AL" "AK" "AZ" "AR" ... $\qquad$

Example
U.S. State Facts and Features. The region in which a
state resides $\qquad$
> state.region [1:4]
[1] South West West South
4 Levels: Northeast South ... West $\qquad$
Levels: Northeast South... West
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## Qualitative Data

- Factors have special status in R
- represented internally by numbers, but not always printed that way
- constructed with factor command
- Displaying Qualitative Data
- first try: make a (contingency) table with table function
- prop.table makes a relative frequency table
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[^0]```
Displaying Qualitative Data
    > Tbl <- table(state.division)
    > Tbl # frequencies
    state.division
        New England Middle Atlantic
        6 3
        South Atlantic East South Central
        8 4
    West South Central East North Central
        4 5
    West North Central Mountain
        7 8
        Pacific
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```

Notes

## Displaying Qualitative Data

```
> Tbl/sum(Tbl) # relative frequencies
state.division
        New England Middle Atlantic
        0.12 0.06
    South Atlantic East South Central
        0.16 0.08
West South Central East North Central
        0.08 0.10
West North Central Mountain
        0.14 0.16
        Pacific
        0.10

\section*{Displaying Qualitative Data}

Notes
> prop.table(Tbl) \# same thing
state.division
New England Middle Atlantic
0.120 .06

South Atlantic East South Central
\(0.16 \quad 0.08\)
West South Central East North Central
0.08
0.10

West North Central Mountain
\(0.14 \quad 0.16\)
Pacific
0.10
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\section*{Bar Graphs}
- discrete analogue of the histogram
- make bar for each level of a factor
- may show frequencies or relative frequencies
- impression given depends on order of bars (default:
\(\qquad\) alphabetical)

\section*{Example}

\section*{U.S. State Facts and Features. State region}

Notes
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\(\qquad\)
> barplot(table(state.region))
> barplot(prop.table(table(state.region)))
\(\qquad\)
\(\qquad\)

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}

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Figure: (Relative) frequency bar graphs of state.region
\(\qquad\)

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\section*{Pareto Diagram}
- a bar graph with ordered bars
- bar with highest (relative) frequency goes on left
- bars drop from left to right
- can sometimes help discern hidden structure

\section*{Example}
U.S. State Facts and Features. State division

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\(\qquad\)
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> library(qcc)
> pareto.chart(table(state.division),
+ ylab = "Frequency")

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}

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Figure: Pareto diagram of state.division
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\(\qquad\)
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Notes
- a bar graph on its side \(\qquad\)
- has dots instead of bars \(\qquad\)
- can show complicated multivariate relationships \(\qquad\)
Example
U.S. State Facts and Features. State region
\(\qquad\)
\(>x<-\) table(state.region)
\(>\) dotchart(as.vector \((x)\), labels \(=\) names \((x)\) ) \(\qquad\)
\(\qquad\)

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\section*{Other Data Types}
```

- Logical
> x <- 5:9
> y<- (x < 7.3)
>
[1] TRUE TRUE TRUE FALSE FALSE
> !y
[1] FALSE FALSE FALSE TRUE TRUE
- Missing


## Other Data Types

Notes

- Missing: represented by NA
$\qquad$
$>x<-c(3,7, N A, 4,7)$ $\qquad$
$>y<-c(5, N A, 1,2,2)$
$>x+y$
[1] 8 NA NA 69
$\qquad$

Some functions have na.rm argument
> is.na(x)
[1] FALSE FALSE TRUE FALSE FALSE
> z <- x[!is.na(x)]
$>\operatorname{sum}(z)$ $\qquad$
[1] 21
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Features of Data Distributions

Four Basic Features
(1) Center: middle or general tendency
(2) Spread: small means tightly clustered, large means highly variable
(3) Shape: symmetry versus skewness, kurtosis
(44 Unusual Features: anything else that pops out at you about the data
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## Symmetry versus Skewness

- symmetric
- right (positive) and left (negative) skewness $\qquad$
$\qquad$


## Kurtosis

- leptokurtic - steep peak, heavy tails
- platykurtic - flatter, thin tails
- mesokurtic - right in the middle
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Notes

```
> stem.leaf(faithful$eruptions)
1 | 2: represents 1.2
    leaf unit: 0.1
        n: 272
    lol
    71 2* | 00000000000011111111
    87 t | 2222222222333333
    92 f f | 44444
    94 s | 66
    97 2. | 889
    98 3* | 0
    102 t | 3333
    108 f | 445555
    118 s | 6666677777
    (16) 3. | 8888888889999999
    138 4* | 000000000000000011111111111111
    lu
    43 s | 6666666666677777777777
    43 rrl 666668666667777777
    4 5* | 0001
- Extreme observation: falls far from the rest of the data
- possible sources
- could be typo
- could be in wrong study
- could be indicative of something deeper
- Quantitatively measure features: Descriptive Statistics
- qualitative data: frequencies or relative frequencies
- quantitative data: measures of CUSS \(\qquad\)
\(\qquad\)
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(

Measures of center: sample mean \(\bar{x}\) (read " \(x\)-bar"):
\[
\begin{equation*}
\bar{x}=\frac{x_{1}+x_{2}+\cdots+x_{n}}{n}=\frac{1}{n} \sum_{i=1}^{n} x_{i} \tag{1}
\end{equation*}
\]

Notes
\(\qquad\)
\(\qquad\)
- Good: natural, easy to compute, nice properties
- Bad: sensitive to extreme values

\section*{How to do it with R}
> stack.loss \# built-in data
[1] \(\begin{array}{lllllllllllll}42 & 37 & 37 & 28 & 18 & 18 & 19 & 20 & 15 & 14 & 14 & 13 & 11\end{array}\)
\(\begin{array}{lllllllll}\text { [14] } & 12 & 8 & 7 & 8 & 8 & 9 & 15 & 15\end{array}\)
> mean(stack.loss)
[1] 17.52381

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}

\section*{How to find it}
(1) sort the data into an increasing sequence of \(n\) numbers
(2) \(\tilde{x}\) lies in position \((n+1) / 2\) \(\qquad\)
- Good: resistant to extreme values, easy to describe
- Bad: not as mathematically tractable, need to sort the
\(\qquad\) data to calculate \(\qquad\)

\section*{How to do it with R}
\(\qquad\)
> median(stack.loss)
[1] 15
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\(\qquad\)
\(\qquad\)
\(\qquad\)

Notes

\section*{How to find it}
\(\qquad\)
(1) "trim" a proportion of data from both ends of the ordered \(\qquad\) list
(2) find the sample mean of what's left
- Good: also resistant to extreme values, has good \(\qquad\) properties, too
- Bad: still need to sort data to get rid of outliers

How to do it with R
```

> mean(stack.loss, trim = 0.05)

```
[1] 16.78947

\section*{Order statistics}

Notes

Given data \(x_{1}, x_{2}, \ldots, x_{n}\), sort in an increasing sequence
\[
\begin{equation*}
x_{(1)} \leq x_{(2)} \leq x_{(3)} \leq \cdots \leq x_{(n)} \tag{2}
\end{equation*}
\]
\(\qquad\)
\(\qquad\)
- \(x_{(k)}\) is the \(k^{\text {th }}\) order statistic
- approx \(100(k / n) \%\) of the observations fall below \(x_{(k)}\)

\section*{How to do it with R}
> sort(stack.loss)
[1] \(\begin{array}{llllllllllllll}7 & 8 & 8 & 8 & 9 & 11 & 12 & 13 & 14 & 14 & 15 & 15 & 15\end{array}\)
[14] \(18 \quad 18 \quad 19 \quad 20 \quad 28 \quad 37 \quad 37 \quad 42\)
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Sample quantile, order \(p(0 \leq p \leq 1)\), denoted \(\tilde{q}_{p}\)
We describe the default (type \(=7\) )
Notes
(1) get the order statistics \(x_{(1)}, x_{(2)}, \ldots, x_{(n)}\).
\(\qquad\)
(2) calculate \((n-1) p+1\), write in form \(k . d\), with \(k\) an
\(\qquad\) integer and \(d\) a decimal
(3)
\[
\begin{equation*}
\tilde{q}_{p}=x_{(k)}+d\left(x_{(k+1)}-x_{(k)}\right) \tag{3}
\end{equation*}
\]
\(\qquad\)
\(\qquad\)
\(\qquad\)
- approximately \(100 p \%\) of the data fall below the value \(\tilde{q}_{p}\).

\section*{How to do it with \(\mathbf{R}\)}
> quantile(stack.loss, probs \(=c(0,0.25,0.37))\)
0\% 25\% 37\%
7.011 .013 .4
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The sample variance \(s^{2}\)
\[
\begin{equation*}
s^{2}=\frac{1}{n-1} \sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2} \tag{4}
\end{equation*}
\]

The sample standard deviation is \(s=\sqrt{s^{2}}\) ．
－Good：tractable，nice mathematical／statistical properties
－Bad：sensitive to extreme values

\section*{How to do it with R}
＞var（stack．loss）；sd（stack．loss）
［1］ 103.4619 \(\qquad\)
［1］ 10.17162 \(\qquad\)
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\section*{Interpretation of s}

\section*{Chebychev＇s Rule：}

The proportion of observations within \(k\) standard deviations of the mean is at least \(1-1 / k^{2}\) ，i．e．，at least \(75 \%, 89 \%\) ，and \(94 \%\) of the data are within 2,3 ，and 4 standard deviations of

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\(\qquad\)
\(\qquad\) the mean，respectively．

\section*{Empirical Rule：}

If data follow a bell－shaped curve，then approximately \(68 \%\) ， \(95 \%\) ，and \(99.7 \%\) of the data are within 1,2 ，and 3 standard deviations of the mean，respectively．

The Interquartile range \(I Q R\)
\[
\begin{equation*}
I Q R=\tilde{q}_{0.75}-\tilde{q}_{0.25} \tag{5}
\end{equation*}
\]
\(\qquad\)
- Good: resistant to outliers
- Bad: only considers middle \(50 \%\) of the data

How to do it with R
> IQR(stack.loss) \(\qquad\)
[1] 8 \(\qquad\)
\(\qquad\)
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Measures of spread: median absolute deviation

The median absolute deviation MAD:
Notes
(1) get the order statistics, find the median \(\tilde{x}\).
(2) calculate the absolute deviations:
\[
\left|x_{1}-\tilde{x}\right|,\left|x_{2}-\tilde{x}\right|, \ldots,\left|x_{n}-\tilde{x}\right|
\]
(3) the \(M A D \propto \operatorname{median}\left\{\left|x_{1}-\tilde{x}\right|,\left|x_{2}-\tilde{x}\right|, \ldots,\left|x_{n}-\tilde{x}\right|\right\}\)
- Good: excellently robust
- Bad: not as popular, not as intuitive

\section*{How to do it with R}
> mad(stack.loss)
[1] 5.9304

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The range \(R\) :
\[
\begin{equation*}
R=x_{(n)}-x_{(1)} \tag{6}
\end{equation*}
\]
\(\qquad\)
\(\qquad\)
- Good (not so much): easy to describe and calculate \(\qquad\)
- Bad: ignores everything but the most extreme observations

\section*{How to do it with R}
> range(stack.loss)
[1] 742
> diff(range(stack.loss))
[1] 35
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Probability and Statistics

Measures of shape: sample skewness
The sample skewness \(g_{1}\) :
\[
\begin{equation*}
g_{1}=\frac{1}{n} \frac{\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{3}}{s^{3}} \tag{7}
\end{equation*}
\]

Things to notice:
- invariant w.r.t. location and scale
- \(-\infty<g_{1}<\infty\)
- sign of \(g_{1}\) indicates direction of skewness \(( \pm)\)

\section*{How to do it with R}
> library(e1071)
> skewness(stack.loss)
[1] 1.156401
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How big is BIG? \(\qquad\)
4.34 versus \(0.434 ? ?\)
\(\qquad\)
\(\qquad\)
Rule of thumb: \(\qquad\)
If \(\left|g_{1}\right|>2 \sqrt{6 / n}\), then the data distribution is substantially \(\qquad\)
skewed (in the direction of the sign of \(g_{1}\) ).
\(\qquad\)
> skewness(discoveries)
[1] 1.207600
> 2 * sqrt(6/length(discoveries))
\(\qquad\)
[1] 0.4898979
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Notes
The sample excess kurtosis \(g_{2}\) : \(\qquad\)
\[
\begin{equation*}
g_{2}=\frac{1}{n} \frac{\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{4}}{s^{4}}-3 \tag{9}
\end{equation*}
\]

Things to note:
- invariant w.r.t. location and scale
- \(-2 \leq g_{2}<\infty\)
- \(g_{2}>0\) indicates leptokurtosis, \(g_{2}<0\) indicates platykurtosis
\(\qquad\)
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\(\qquad\)
\(\qquad\)

\section*{How to do it with \(\mathbf{R}\)}
> library(e1071)
> kurtosis(stack.loss)
[1] 0.1343524
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Again, how big is BIG?

\section*{Rule of thumb:}

If \(\left|g_{2}\right|>4 \sqrt{6 / n}\), then the data distribution is substantially kurtic.
\(\qquad\)
> kurtosis(UKDriverDeaths) \(\qquad\)
[1] 0.07133848 \(\qquad\)
> 4 * sqrt(6/length(UKDriverDeaths)) \(\qquad\)
[1] 0.7071068 \(\qquad\)
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Notes
Exploratory data analysis: more on stemplots
- Trim Outliers: observations that fall far from the bulk of the other data often obscure structure to the data and are
\(\qquad\) best left out. Use the trim. outliers argument to \(\qquad\) stem.leaf.
- Split Stems: we sometimes fix "skyscraper" stemplots by increasing the number of lines available for a given stem. The end result is a more spread out stemplot which often looks better. Use the \(m\) argument to stem.leaf
- Depths: give insight into balance of the data around the \(\qquad\) median. Frequencies are accumulated from the outside inward, including outliers. Use depths = TRUE.

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\section*{More about stemplots}

\section*{> stem.leaf(faithful\$eruptions)}
```

1 | 2: represents 1.2
leaf unit: 0.1
s | 667777777777

```

```

    1. 8888888888888888888
    | |0000000000011111
    t 2222222
        f | 44444
        s 5 }
        3*1 0
        t | 3333
        f | 445555
        s | 6666677777
        3.। }888888888999999
    4* | 0000000000000000111111111111111
    t | 22222222222233333333333333333
    f | 44444444444445555555555555555555555
    s | 6666666666677777777777
    4. | 88888888888899999
    5* | 0001
    ```

```

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    ```

Hinges and the 5NS
- Find the order statistics \(x_{(1)}, x_{(2)}, \ldots, x_{(n)}\).
- The lower hinge \(h_{L}\) is in position \(L=\lfloor(n+3) / 2\rfloor / 2\)
- The upper hinge \(h_{U}\) is in position \(n+1-L\).

Given the hinges, the five number summary (5NS) is
\[
\begin{equation*}
5 N S=\left(x_{(1)}, h_{L}, \tilde{x}, h_{U}, x_{(n)}\right) . \tag{10}
\end{equation*}
\]

How to do it with R
> fivenum(stack.loss)
[1] 711151942

Boxplot: a visual display of the 5NS. Can visually assess multiple features of the data set:
- Center: estimated by the sample median, \(\tilde{x}\)
\(\qquad\)
- Spread: judged by the width of the box, \(h_{U}-h_{L}\) \(\qquad\)
- Shape: indicated by the relative lengths of the whiskers, \(\qquad\) position of the median inside box. \(\qquad\)
- Extreme observations: identified by open circles

\section*{How to do it with \(\mathbf{R}\)}
```

> boxplot(rivers, horizontal = TRUE)

```
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Figure: Boxplot of rivers
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Standardizing variables
- useful to see how observation relates to other observations
- AKA measure of relative standing, \(z\)-score
\[
z_{i}=\frac{x_{i}-\bar{x}}{s}, \quad i=1,2, \ldots, n
\]
- unitless
- positive (negative) \(z\)-score falls above (below) mean

\section*{How to do it with R}
> scale(precip) [1:3]
[1] \(2.342971 \quad 1.445597-2.034466\)
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- usually have two (or more) measurements associated with \(\qquad\) each subject
- display in rectangular array \(\qquad\)
- each row corresponds to a subject \(\qquad\)
- columns contain the measurements for each variable
\(\qquad\)
How to do it with R
\(>x<-5: 6 ; y<-\) letters \([3: 4] ; z<-c(0.1,3.8)\)
> data.frame (v1 = x, v2 = y, v3 = z)
v1 v2 v3
15 c 0.1
26 d 3.8
\(\qquad\)
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\section*{More on data frames}
- must have same number of rows in each column
- all measurements in single column must be same type
- indexing is two-dimensional; the columns have names

\section*{How to do it with R}
\(>A<-\) data.frame (v1 = x, v2 = y, v3 = z)
\(>A[2,1] ; A[1,] ; A[, 3]\)
[1] 6
v1 v2 v3
15 c 0.1
[1] 0.13 .8

\section*{Two categorical variables}
\(\qquad\)
- usually make a two-way contingency table \(\qquad\)
- in the R Commander with Statistics \(\triangleright\) Contingency Tables \(\qquad\)
\(\triangleright\) Two-way Tables
How to do it with R
> library (RcmdrPlugin.IPSUR)
> data(RcmdrTestDrive)
> xtabs( \(\sim\) gender + smoking, data \(=\) RcmdrTestDrive) \(\qquad\)
smoking
gender Nonsmoker Smoker
\begin{tabular}{lrr} 
Female & 61 & 9 \\
Male & 75 & 23
\end{tabular}
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\section*{Bivariate data: more on tables}
- Descriptive statistics: for now, marginal totals/percentages
more to talk about later: odds ratio, relative risk
\(\qquad\)
```

How to do it with R
> A <- xtabs(Freq ~ Survived + Class, data = Titanic)
> addmargins(A)
Class
Survived 1st 2nd 3rd Crew Sum

| No | 122 | 167 | 528 | 673 | 1490 |
| :--- | :--- | :--- | :--- | :--- | :--- |


| Yes | 203 | 118 | 178 | 212 | 711 |
| :--- | :--- | :--- | :--- | :--- | :--- |

    Sum 325 285 706 885 2201
    ```
\(\qquad\)

\section*{Bivariate data: more on tables}
```

> library(abind)
> colPercents(A)
Class
Survived 1st 2nd 3rd Crew

```

```

    Yes }\quad\begin{array}{lllll}{62.5}&{41.4}&{25.2}&{24}
    Total 100.0 100.0 100.0 100
    Count 325.0 285.0 706.0 885
    > rowPercents(A)
Class
Survived 1st 2nd 3rd Crew Total Count
No 8.2 11.2 35.4 45.2 100 1490

```

```

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    ```

\section*{Plotting two categorical variables}
- Stacked bar charts
- Side-by-side bar charts
- Spine plots

How to do it with R \(\qquad\)
```

> barplot(A, legend.text = TRUE)
> barplot(A, legend.text = TRUE, beside = TRUE)
> spineplot(A)

```

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Figure: Stacked bar chart of Titanic data
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\(\qquad\)
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Figure: Side-by-side bar chart of Titanic data

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Figure: Spine plot of Titanic data
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Bivariate data: quantitative versus quantitative
- Can do univariate graphs of both variables separately
- Make scatterplots for both variables simultaneously

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> library(lattice)
> xyplot(conc ~ rate, data = Puromycin) \(\qquad\)

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Figure: Scatterplot of Puromycin data
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Figure: Scatterplot of Puromycin data \(\qquad\)

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Figure: Scatterplot of attenu data
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Figure: Scatterplot of faithful data \(\qquad\)

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Figure: Scatterplot of iris data
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Figure: Scatterplot of iris data

\section*{Measuring Linear association}

Notes
The sample Pearson product-moment correlation coefficient:
\[
r=\frac{\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)\left(y_{i}-\bar{y}\right)}{\sqrt{\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)} \sqrt{\sum_{i=1}^{n}\left(y_{i}-\bar{y}\right)}}
\]
\(\qquad\)
- independent of scale \(\qquad\)
- \(-1 \leq r \leq 1\), equality when points lie on straight line \(\qquad\)

\section*{How to do it with R}
> with(iris, cor(Petal.Width, Petal.Length))
\(\qquad\)
[1] 0.9628654
> with(attenu, cor(dist, accel))
[1] -0.4713809
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More about linear correlation
- measures strength and direction of linear association \(\qquad\)
- Rules of thumb:
- \(0<|r|<0.3\), weak linear association
- \(0.3<|r|<0.7\), moderate linear association
- \(0.7<|r|<1\), strong linear association \(\qquad\)
- Just because \(r \approx 0\) doesn't mean there isn't any association \(\qquad\)
\(\qquad\)
- Break down quantitative var by groups of subjects \(\qquad\)
- compare centers and spreads: variation within versus \(\qquad\) between groups
- compare clusters and gaps \(\qquad\)
- compare outliers and unusual features \(\qquad\)
- compare shapes. \(\qquad\)
- graphical and numerical
\(\qquad\)
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Notes

\section*{Comparison of groups}

\section*{How to do it with R}
> stripchart(weight ~ feed, method = "stack",
+ data = chickwts)
> library(lattice)
> histogram(~age | education, data = infert)
> bwplot(~count | spray, data = InsectSprays)
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Figure: Stripcharts of chickwts data
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Figure: Histograms of infert data

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Figure: Boxplots of InsectSprays data
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\section*{Multiple variables}

With more variables, complexity increases
Notes
- multi-way contingency tables (bunch of categorical vars)
\(\qquad\)
- mosaic plots, dotcharts
- sample variance-covariance matrices
- scatterplot matrices
- comparing groups: coplots

\section*{How to do it with R}
> splom(~cbind(Murder, Assault, Rape),
\(+\quad\) data \(=\) USArrests)
> '?'(dotchart)
> '?`(xyplot)
> '?'(mosaicplot)


Scatter Plot Matrix

Figure: Scatterplot matrix of LifeCycleSavings data

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Class

Figure: Mosaic plot of Titanic data

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Notes

Figure: Shingle plot of Titanic data

Notes```


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