# STAT 3743: Probability and Statistics

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# Types of Data

datum: any piece of information data set: collection of data related to each other somehow

- Categories of Data
  - quantitative: associated with a measurement of some quantity on an observational unit,
  - qualitative: associated with some quality or property of the observational unit,
  - logical: represents true/false, important later
  - missing: should be there but aren't
  - other types: everything else

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### Quantitative Data

Quantitative data: any that measure the quantity of something

- invariably assume numerical values
- can be further subdivided:
  - Discrete data take values in a finite or countably infinite set of numbers
  - Continuous data take values in an interval of numbers.

    AKA scale, interval, measurement
- distinction between discrete and continuous data not always clear-cut

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### Example

**Annual Precipitation in US Cities.** (precip) avg amount rainfall (in.) for 70 cities in US and Puerto Rico.

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> str(precip)

Named num [1:70] 67 54.7 7 48.5 14 17.2 20.7 13 43.4 40.2 ...

- attr(\*, "names")= chr [1:70] "Mobile" "Juneau" "Phoenix" "Little Rock" ...

> precip[1:4]

Mobile Juneau Phoenix 67.0 54.7 7.0

Little Rock

48.5

quantitative, continuous

### Example

### **Lengths of Major North American Rivers.** (rivers)

lengths (mi) of rivers in North America. See ?rivers.

```
> str(rivers)
num [1:141] 735 320 325 392 524 ...
> rivers[1:4]
[1] 735 320 325 392
```

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### Example

### Yearly Numbers of Important Discoveries.

(discoveries) numbers of "great" inventions/discoveries in each year from 1860 to 1959 (from 1975 World Almanac)

> str(discoveries)

Time-Series [1:100] from 1860 to 1959: 5 3 0 2 0 3 2 3 6 1 ...

> discoveries[1:4]

[1] 5 3 0 2

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# Displaying Quantitative Data

- Strip charts (or Dot plots):
  - for either discrete or continuous data
  - usually best when data not too large.
- the stripchart function
  - three methods
    - overplot only distinct values
    - jitter add noise in y direction
    - stack repeats on top of one another

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# Displaying Quantitative Data

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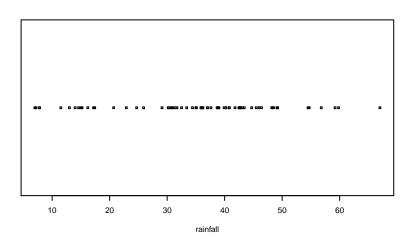


Figure: Stripchart of precip

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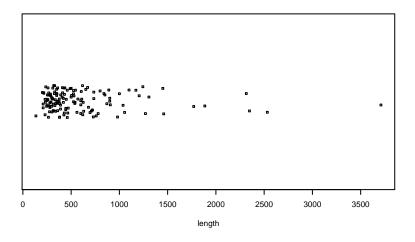


Figure: Stripchart of rivers

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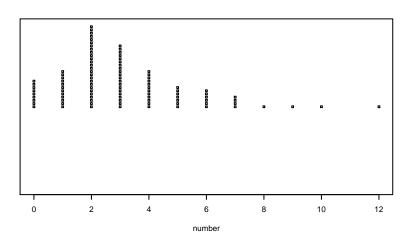


Figure: Stripchart of discoveries

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# Histograms

### Histograms

- typically for continuous data
- decide on bins/classes, make bars proportional to membership
- often misidentified (bar graphs)
- > hist(precip, main = "")
- > hist(precip, freq = FALSE, main = "")

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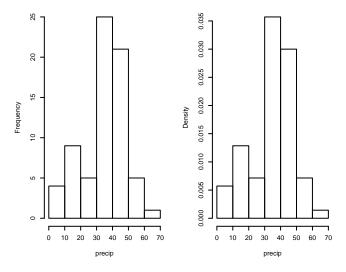


Figure: Histograms of precip

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## Remarks about histograms

- choose different bins, get a different histogram
- many algorithms for choosing bins automatically
- should investigate several bin choices
  - look for stability
  - try to capture underlying story of data

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# Stemplots

- Stemplots have two basic parts: stems and leaves
  - initial digit(s) taken for stem
  - trailing digits stand for leaves
  - leaves accumulate to the right

### Example

Road Casualties in Great Britain 1969-84. A time series of total car drivers killed or seriously injured in Great Britain monthly from Jan 1969 to Dec 1984.

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# Stemplot of UK Driver Deaths

```
> library(aplpack)
> stem.leaf(UKDriverDeaths, depth = FALSE)
1 | 2: represents 120
leaf unit: 10
  10 | 57
  11 | 136678
  12 | 123889
  13 | 0255666888899
  14 | 00001222344444555556667788889
  15 | 0000111112222223444455555566677779
  16 | 01222333444445555555678888889
  17 | 11233344566667799
  18 | 00011235568
  19 | 01234455667799
  20 | 0000113557788899
  21 | 145599
  22 | 013467
  23 | 9
  24 | 7
HI: 2654
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```

## Code for stemplots

17 | 11233344566667799

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### Index Plots

Good for plotting data ordered in time

- a 2-D plot, with index (observation number) on x-axis,
   value on y-axis
- two methods
  - spikes: draws vertical line up to value (type = "h")
  - points: simple dot at the observed height (type = "p")

### Example

**Level of Lake Huron 1875-1972.** annual measurements of the level (in feet) of Lake Huron from 1875–1972.

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### Index Plots

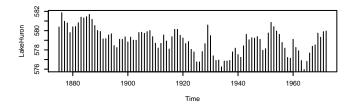
Good for plotting data ordered in time

- a 2-D plot, with index (observation number) on x-axis,
   value on y-axis
- two methods
  - spikes: draws vertical line up to value (type = "h")
  - points: simple dot at the observed height (type = "p")

### Example

**Level of Lake Huron 1875-1972.** annual measurements of the level (in feet) of Lake Huron from 1875–1972.

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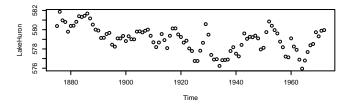


Figure: Index plots of LakeHuron

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# Qualitative Data, Categorical Data, Factors

- Qualitative data: any data that are not numerical, or do not represent numerical quantities
  - some data *look* qualitative. Example: shoe size
  - some data identify the observation, not of much interest
- Factors subdivide data into categories
  - possible values of a factor: levels
  - factors may be *nominal* or *ordinal* 
    - nominal: levels are names, only (gender, political party ethnicity)
    - ordinal: levels are ordered (SES, class rank, shoe size)

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### Example

U.S. State Facts and Features. postal abbreviations

```
> str(state.abb)
chr [1:50] "AL" "AK" "AZ" "AR" ...
```

### Example

U.S. State Facts and Features. The region in which a state resides

```
> state.region[1:4]
[1] South West West South
4 Levels: Northeast South ... West
```

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## Qualitative Data

- Factors have special status in R
  - represented internally by numbers, but not always printed that way
  - constructed with factor command
- Displaying Qualitative Data
  - first try: make a (contingency) table with table function
  - prop.table makes a relative frequency table

### Example

U.S. State Facts and Features. State division

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# Displaying Qualitative Data

```
> Tbl <- table(state.division)
> Tbl  # frequencies
state.division

New England  Middle Atlantic
6  3
South Atlantic East South Central
8  4
West South Central East North Central
4  5
West North Central  Mountain
7  8
Pacific
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```

# Displaying Qualitative Data

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> Tbl/sum(Tbl)	<pre># relative frequencies</pre>	
state.division		
New England	Middle Atlantic	
0.12	0.06	
South Atlantic	East South Central	
0.16	0.08	
West South Central	East North Central	
0.08	0.10	
West North Central	Mountain	
0.14	0.16	
Pacific		
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## Displaying Qualitative Data

# same thing > prop.table(Tbl) state.division New England Middle Atlantic 0.12 0.06 South Atlantic East South Central 0.16 0.08 West South Central East North Central 0.08 0.10 West North Central Mountain 0.16 0.14 Pacific 0.10 G. Jay Kerns, Youngstown State University Probability and Statistics

Bar Graphs

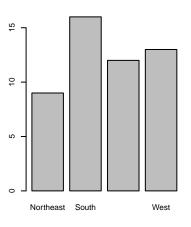
- discrete analogue of the histogram
- make bar for each level of a factor
- may show frequencies or relative frequencies
- impression given depends on order of bars (default: alphabetical)

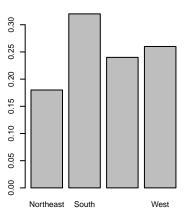
Example

U.S. State Facts and Features. State region

- > barplot(table(state.region))
- > barplot(prop.table(table(state.region)))

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Figure: (Relative) frequency bar graphs of state.region

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# Pareto Diagram

- a bar graph with ordered bars
- bar with highest (relative) frequency goes on left
- bars drop from left to right
- can sometimes help discern hidden structure

### Example

### U.S. State Facts and Features. State division

- > library(qcc)
- > pareto.chart(table(state.division),
- + ylab = "Frequency")

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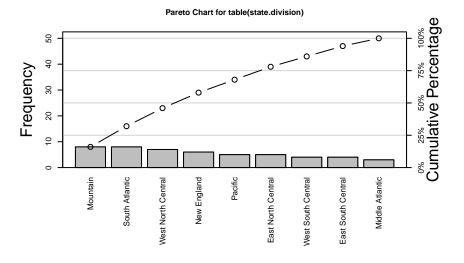


Figure: Pareto diagram of state.division

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## **Dot Charts**

- a bar graph on its side
- has dots instead of bars
- can show complicated multivariate relationships

### Example

### U.S. State Facts and Features. State region

- > x <- table(state.region)
- > dotchart(as.vector(x), labels = names(x))

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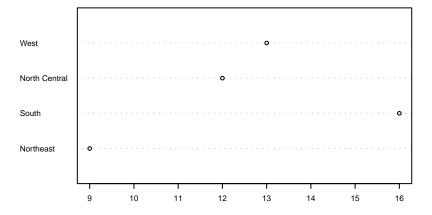


Figure: Dot chart of state.region

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# Other Data Types

### Logical

$$> y \leftarrow (x < 7.3)$$

> y

[1] TRUE TRUE TRUE FALSE FALSE

> !y

[1] FALSE FALSE FALSE TRUE TRUE

Missing

Notes

## Other Data Types

Missing: represented by NA

> 
$$x \leftarrow c(3, 7, NA, 4, 7)$$
  
>  $y \leftarrow c(5, NA, 1, 2, 2)$   
>  $x + y$   
[1] 8 NA NA 6 9

• Some functions have na.rm argument

[1] FALSE FALSE TRUE FALSE FALSE

> sum(z)

[1] 21

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### Features of Data Distributions

Four Basic Features

- 1 Center: middle or general tendency
- 2 Spread: small means tightly clustered, large means highly variable
- 3 Shape: symmetry versus skewness, kurtosis
- Unusual Features: anything else that pops out at you about the data

Votes			

# More about shape

### Symmetry versus Skewness

- symmetric
- right (positive) and left (negative) skewness

### **Kurtosis**

- leptokurtic steep peak, heavy tails
- platykurtic flatter, thin tails
- mesokurtic right in the middle

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# Unusual features: clusters or gaps

> stem.leaf(faithful\$eruptions)

```
1 | 2: represents 1.2
leaf unit: 0.1
       n: 272
 12 s | 66777777777
 71 2* | 0000000000011111111
 87 t | 222222222333333
 92 f | 44444
 94 s | 66
 97 2. | 889
     3* | 0
      t | 3333
      f | 445555
 118 s | 6666677777
 (16) 3. | 888888889999999
 138 4* | 00000000000000111111111111111
 107 t | 22222222222333333333333333333
 43 s | 66666666667777777777
 21 4. | 888888888899999
  4 5* | 0001
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```

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### Unusual features: extreme observations

- Extreme observation: falls far from the rest of the data
  - possible sources
    - could be typo
    - could be in wrong study
    - could be indicative of something deeper
- Quantitatively measure features: Descriptive Statistics
  - qualitative data: frequencies or relative frequencies
  - quantitative data: measures of CUSS

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# Measures of center: sample mean $\overline{x}$ (read "x-bar"):

$$\overline{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{1}{n} \sum_{i=1}^n x_i.$$
 (1)

- Good: natural, easy to compute, nice properties
- Bad: sensitive to extreme values

#### How to do it with R

> stack.loss # built-in data

[1] 42 37 37 28 18 18 19 20 15 14 14 13 11

[14] 12 8 7 8 8 9 15 15

> mean(stack.loss)

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[1]	17.52381	

## Measures of center: sample median $\tilde{x}$

#### How to find it

- $\bigcirc$  sort the data into an increasing sequence of n numbers
- ②  $\tilde{x}$  lies in position (n+1)/2
- Good: resistant to extreme values, easy to describe
- Bad: not as mathematically tractable, need to sort the data to calculate

#### How to do it with R

> median(stack.loss)

[1] 15

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## Measures of center: trimmed mean $\overline{x}_t$

#### How to find it

- "trim" a proportion of data from both ends of the ordered list
- 2 find the sample mean of what's left
- Good: also resistant to extreme values, has good properties, too
- Bad: still need to sort data to get rid of outliers

### How to do it with R

> mean(stack.loss, trim = 0.05)

[1] 16.78947

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### Order statistics

Given data  $x_1, x_2, \ldots, x_n$ , sort in an increasing sequence

$$x_{(1)} \le x_{(2)} \le x_{(3)} \le \cdots \le x_{(n)}$$
 (2)

- $x_{(k)}$  is the  $k^{th}$  order statistic
- approx 100(k/n)% of the observations fall below  $x_{(k)}$

### How to do it with R

> sort(stack.loss)

[1] 7 8 8 8 9 11 12 13 14 14 15 15 15

[14] 18 18 19 20 28 37 37 42

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# Sample quantile, order p ( $0 \le p \le 1$ ), denoted $\tilde{q}_p$

We describe the default (type = 7)

- ① get the order statistics  $x_{(1)}, x_{(2)}, \ldots, x_{(n)}$ .
- ② calculate (n-1)p+1, write in form k.d, with k an integer and d a decimal

3

$$\tilde{q}_p = x_{(k)} + d(x_{(k+1)} - x_{(k)}).$$
 (3)

ullet approximately 100p% of the data fall below the value  $ilde{q}_p$  .

#### How to do it with R

> quantile(stack.loss, probs = c(0, 0.25, 0.37))
0% 25% 37%

7.0 11.0 13.4

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# Measures of spread: sample variance, std. deviation

The sample variance  $s^2$ 

$$s^{2} = \frac{1}{n-1} \sum_{i=1}^{n} (x_{i} - \overline{x})^{2}$$
 (4)

The sample standard deviation is  $s = \sqrt{s^2}$ .

- Good: tractable, nice mathematical/statistical properties
- Bad: sensitive to extreme values

#### How to do it with R

> var(stack.loss); sd(stack.loss)

- [1] 103.4619
- [1] 10.17162

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### Interpretation of s

### Chebychev's Rule:

The proportion of observations within k standard deviations of the mean is at least  $1 - 1/k^2$ , *i.e.*, at least 75%, 89%, and 94% of the data are within 2, 3, and 4 standard deviations of the mean, respectively.

### **Empirical Rule:**

If data follow a bell-shaped curve, then approximately 68%, 95%, and 99.7% of the data are within 1, 2, and 3 standard deviations of the mean, respectively.

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# Measures of spread: interquartile range

The Interquartile range IQR

$$IQR = \tilde{q}_{0.75} - \tilde{q}_{0.25} \tag{5}$$

- Good: resistant to outliers
- Bad: only considers middle 50% of the data

#### How to do it with R

> IQR(stack.loss)

[1] 8

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## Measures of spread: median absolute deviation

The median absolute deviation MAD:

- ① get the order statistics, find the median  $\tilde{x}$ .
- 2 calculate the absolute deviations:

$$|x_1 - \tilde{x}|, |x_2 - \tilde{x}|, \dots, |x_n - \tilde{x}|$$

- 3 the  $MAD \propto \text{median} \{ |x_1 \tilde{x}|, |x_2 \tilde{x}|, \dots, |x_n \tilde{x}| \}$
- Good: excellently robust
- Bad: not as popular, not as intuitive

### How to do it with R

> mad(stack.loss)

[1] 5.9304

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# Measures of spread: the range

The *range R*:

$$R = x_{(n)} - x_{(1)} \tag{6}$$

- Good (not so much): easy to describe and calculate
- Bad: ignores everything but the most extreme observations

#### How to do it with R

> range(stack.loss)

[1] 7 42

> diff(range(stack.loss))

[1] 35

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## Measures of shape: sample skewness

The sample skewness  $g_1$ :

$$g_1 = \frac{1}{n} \frac{\sum_{i=1}^{n} (x_i - \overline{x})^3}{s^3}.$$
 (7)

Things to notice:

- invariant w.r.t. location and scale
- $\bullet$   $-\infty < g_1 < \infty$
- sign of  $g_1$  indicates direction of skewness  $(\pm)$

### How to do it with R

- > library(e1071)
- > skewness(stack.loss)

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# Measures of shape: sample skewness

How big is BIG?

4.34 versus 0.434?? (8)

#### Rule of thumb:

If  $|g_1| > 2\sqrt{6/n}$ , then the data distribution is substantially skewed (in the direction of the sign of  $g_1$ ).

- > skewness(discoveries)
- [1] 1.207600
- > 2 \* sqrt(6/length(discoveries))
- [1] 0.4898979

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## Measures of shape: sample excess kurtosis

The sample excess kurtosis  $g_2$ :

$$g_2 = \frac{1}{n} \frac{\sum_{i=1}^{n} (x_i - \overline{x})^4}{s^4} - 3.$$
 (9)

Things to note:

- invariant w.r.t. location and scale
- $\bullet$   $-2 \le g_2 < \infty$
- $g_2 > 0$  indicates leptokurtosis,  $g_2 < 0$  indicates platykurtosis

#### How to do it with R

- > library(e1071)
- > kurtosis(stack.loss)

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## Measures of shape: sample excess kurtosis

Again, how big is BIG?

#### Rule of thumb:

If  $|g_2| > 4\sqrt{6/n}$ , then the data distribution is substantially kurtic.

> kurtosis(UKDriverDeaths)

[1] 0.07133848

> 4 \* sqrt(6/length(UKDriverDeaths))

[1] 0.7071068

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## Exploratory data analysis: more on stemplots

- Trim Outliers: observations that fall far from the bulk of the other data often obscure structure to the data and are best left out. Use the trim.outliers argument to stem.leaf.
- Split Stems: we sometimes fix "skyscraper" stemplots by increasing the number of lines available for a given stem.
   The end result is a more spread out stemplot which often looks better. Use the m argument to stem.leaf
- Depths: give insight into balance of the data around the median. Frequencies are accumulated from the outside inward, including outliers. Use depths = TRUE.

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## More about stemplots

### > stem.leaf(faithful\$eruptions)

```
1 | 2: represents 1.2
leaf unit: 0.1
       s | 66777777777
      1. | 8888888888888888888888889999999999
       2* | 0000000000011111111
       t | 22222222333333
       f | 44444
      s | 66
  97 2. | 889
      3* | 0
 102
       t | 3333
       f | 445555
 118
       s | 6666677777
 (16) 3. | 888888889999999
       4* | 0000000000000001111111111111111
       t | 22222222222333333333333333333
       s | 666666666677777777777
      4. | 888888888899999
       5* | 0001
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```

# Hinges and the 5*NS*

- Find the order statistics  $x_{(1)}$ ,  $x_{(2)}$ , ...,  $x_{(n)}$ .
- The *lower hinge*  $h_L$  is in position  $L = \lfloor (n+3)/2 \rfloor /2$
- The *upper hinge*  $h_U$  is in position n + 1 L.

Given the hinges, the *five number summary* (5NS) is

$$5NS = (x_{(1)}, h_L, \tilde{x}, h_U, x_{(n)}). \tag{10}$$

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#### How to do it with R

> fivenum(stack.loss)

[1] 7 11 15 19 42

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## **Boxplots**

**Boxplot:** a visual display of the 5NS. Can visually assess multiple features of the data set:

- Center: estimated by the sample median,  $\tilde{x}$
- Spread: judged by the width of the box,  $h_U h_L$
- *Shape:* indicated by the relative lengths of the whiskers, position of the median inside box.
- Extreme observations: identified by open circles

#### How to do it with R

> boxplot(rivers, horizontal = TRUE)

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#### 

## Outliers

• potential: falls beyond 1.5 times the width of the box

less than  $h_L - 1.5(h_U - h_L)$  or greater than  $h_U + 1.5(h_U - h_L)$ 

• suspected: falls beyond 3 times the width of the box

less than  $h_L - 3(h_U - h_L)$  or greater than  $h_U + 3(h_U - h_L)$ 

### How to do it with R

- > boxplot.stats(rivers)\$out
- [1] 1459 1450 1243 2348 3710 2315 2533 1306
- [9] 1270 1885 1770

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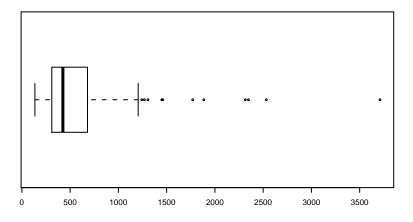


Figure: Boxplot of rivers

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# Standardizing variables

- useful to see how observation relates to other observations
- AKA measure of relative standing, z-score

$$z_i = \frac{x_i - \overline{x}}{s}, \quad i = 1, 2, \dots, n$$

- unitless
- positive (negative) z-score falls above (below) mean

### How to do it with R

> scale(precip)[1:3]

[1] 2.342971 1.445597 -2.034466

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### Multivariate data: data frames

- usually have two (or more) measurements associated with each subject
- display in rectangular array
  - each row corresponds to a subject
  - columns contain the measurements for each variable

#### How to do it with R

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### More on data frames

- must have same number of rows in each column
- all measurements in single column must be same type
- indexing is two-dimensional; the columns have names

#### How to do it with R

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## Bivariate data: qualitative versus qualitative

### Two categorical variables

- usually make a two-way contingency table
- in the R Commander with Statistics ▷ Contingency Tables

#### How to do it with R

- > library(RcmdrPlugin.IPSUR)
- > data(RcmdrTestDrive)
- > xtabs(~ gender + smoking, data = RcmdrTestDrive) smoking

Nonsmoker Smoker gender

61 9 Female

Male 75 23

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### Bivariate data: more on tables

- Descriptive statistics: for now, marginal totals/percentages
- more to talk about later: odds ratio, relative risk

### How to do it with R

- > A <- xtabs(Freq ~ Survived + Class, data = Titanic)
- > addmargins(A)

Class

Survived 1st 2nd 3rd Crew Sum 122 167 528 673 1490 No 203 118 178 212 711 Yes 285 706 885 2201 Sum

Notes			

### Bivariate data: more on tables

```
> library(abind)
> colPercents(A)
       Class
Survived
          1st
                2nd
                      3rd Crew
         37.5 58.6 74.8
  No
                            76
         62.5 41.4 25.2
  Yes
                            24
  Total 100.0 100.0 100.0 100
  Count 325.0 285.0 706.0 885
> rowPercents(A)
       Class
Survived 1st 2nd 3rd Crew Total Count
         8.2 11.2 35.4 45.2
                                   1490
                              100
                                   711
    Yes 28.6 16.6 25.0 29.8
```

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# Plotting two categorical variables

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- Stacked bar charts
- Side-by-side bar charts
- Spine plots

### How to do it with R

```
> barplot(A, legend.text = TRUE)
> barplot(A, legend.text = TRUE, beside = TRUE)
> spineplot(A)
```

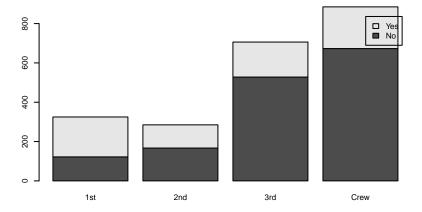


Figure: Stacked bar chart of Titanic data

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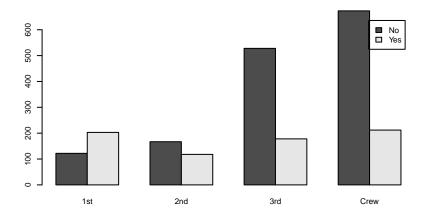


Figure: Side-by-side bar chart of Titanic data

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Figure: Side-by-side bar	cnart of	litanic da
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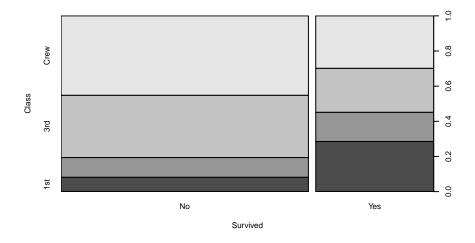


Figure: Spine plot of Titanic data

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# Bivariate data: quantitative versus

- Can do univariate graphs of both variable
- Make scatterplots for both variables simple

### How to do it with R

- > plot(conc ~ rate, data = Puromyci
- > library(lattice)
- > xyplot(conc ~ rate, data = Puromy

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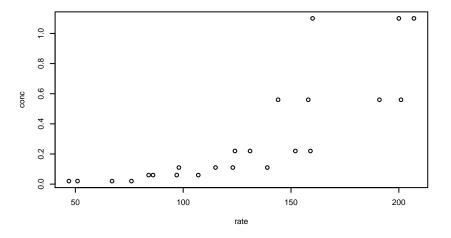


Figure: Scatterplot of Puromycin data

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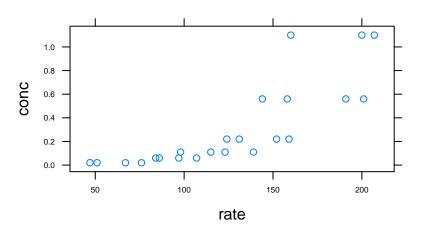


Figure: Scatterplot of Puromycin data

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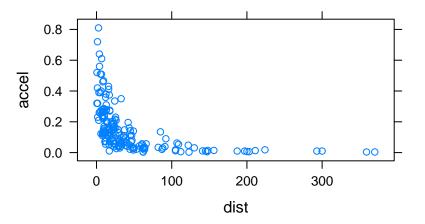


Figure: Scatterplot of attenu data

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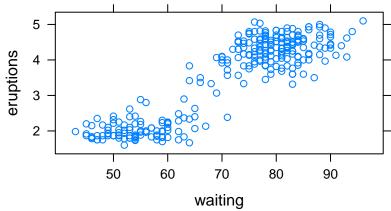


Figure: Scatterplot of faithful data

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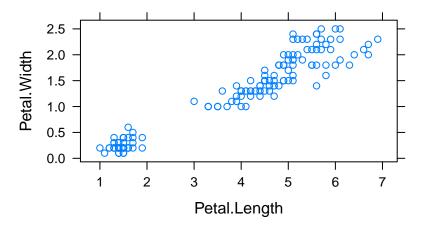


Figure: Scatterplot of iris data

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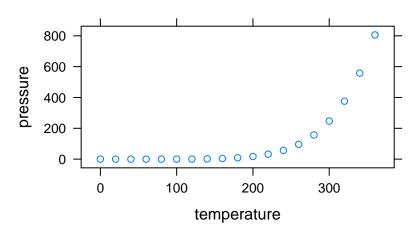


Figure: Scatterplot of iris data

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# Measuring Linear association

### The sample Pearson product-moment correlation coefficient:

$$r = \frac{\sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \overline{x})} \sqrt{\sum_{i=1}^{n} (y_i - \overline{y})}}$$

- independent of scale
- $-1 \le r \le 1$ , equality when points lie on straight line

### How to do it with R

> with(iris, cor(Petal.Width, Petal.Length))

[1] 0.9628654

> with(attenu, cor(dist, accel))

[1] -0.4713809

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### More about linear correlation

- measures strength and direction of linear association
- Rules of thumb:
  - 0 < |r| < 0.3, weak linear association
  - 0.3 < |r| < 0.7, moderate linear association
  - 0.7 < |r| < 1, strong linear association
- Just because  $r \approx 0$  doesn't mean there isn't any association

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## One quantitative, one categorical

- Break down quantitative var by groups of subjects
  - compare centers and spreads: variation within versus between groups
  - compare clusters and gaps
  - compare outliers and unusual features
  - compare shapes.
- graphical and numerical

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## Comparison of groups

#### How to do it with R

- > stripchart(weight ~ feed, method = "stack",
- + data = chickwts)
- > library(lattice)
- > histogram(~age | education, data = infert)
- > bwplot(~count | spray, data = InsectSprays)

-		
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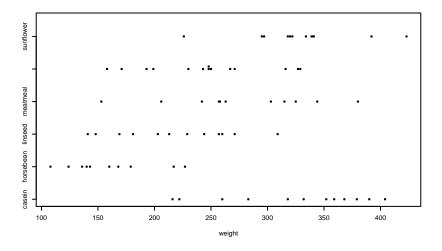


Figure: Stripcharts of chickwts data

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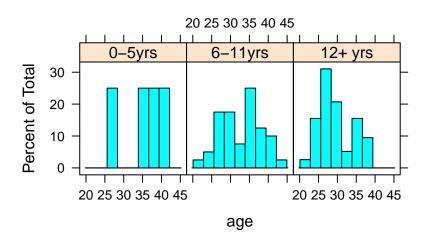


Figure: Histograms of infert data

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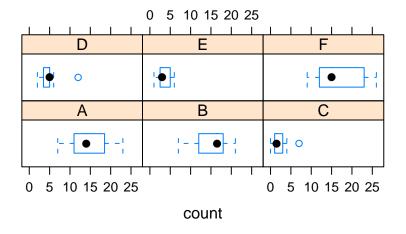


Figure: Boxplots of InsectSprays data

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# Multiple variables

With more variables, complexity increases

- multi-way contingency tables (bunch of categorical vars)
  - mosaic plots, dotcharts
- sample variance-covariance matrices
  - scatterplot matrices
- comparing groups: coplots

### How to do it with R

- > splom(~cbind(Murder, Assault, Rape),
- + data = USArrests)
- > `?`(dotchart)
- > `?`(xyplot)
- > `?`(mosaicplot)

Votes			
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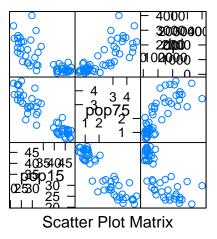


Figure: Scatterplot matrix of LifeCycleSavings data

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Titanic

Titanic

Titanic

Figure: Mosaic plot of Titanic data

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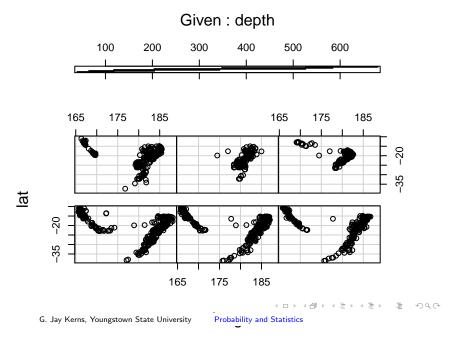


Figure: Shingle plot of Titanic data

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