STAT 3743: Probability and Statistics

G. Jay Kerns, Youngstown State University

Fall 2010

Types of Data

datum: any piece of information

data set: collection of data related to each other somehow

- Categories of Data
 - quantitative: associated with a measurement of some quantity on an observational unit,
 - qualitative: associated with some quality or property of the observational unit,
 - logical: represents true/false, important later
 - missing: should be there but aren't
 - other types: everything else



Quantitative Data

Quantitative data: any that measure the quantity of something

- invariably assume numerical values
- can be further subdivided:
 - Discrete data take values in a finite or countably infinite set of numbers
 - Continuous data take values in an interval of numbers.
 AKA scale, interval, measurement
- distinction between discrete and continuous data not always clear-cut



Annual Precipitation in US Cities. (precip) avg amount rainfall (in.) for 70 cities in US and Puerto Rico.

```
> str(precip)
```

```
Named num [1:70] 67 54.7 7 48.5 14 17.2 20.7 13 43.4 4 - attr(*, "names") = chr [1:70] "Mobile" "Juneau" "Phoe
```

> precip[1:4]

Phoenix	Juneau	Mobile
7.0	54.7	67.0

Little Rock

48.5

Lengths of Major North American Rivers. (rivers) lengths (mi) of rivers in North America. See ?rivers.

```
> str(rivers)
num [1:141] 735 320 325 392 524 ...
> rivers[1:4]
[1] 735 320 325 392
```

Yearly Numbers of Important Discoveries.

(discoveries) numbers of "great" inventions/discoveries in each year from 1860 to 1959 (from 1975 World Almanac)

```
> str(discoveries)
```

```
Time-Series [1:100] from 1860 to 1959: 5 3 0 2 0 3 2 3
```

> discoveries[1:4]

```
[1] 5 3 0 2
```

Displaying Quantitative Data

- Strip charts (or Dot plots):
 - for either discrete or continuous data
 - usually best when data not too large.
- the stripchart function
 - three methods:
 - overplot only distinct values
 - jitter add noise in y direction
 - stack repeats on top of one another

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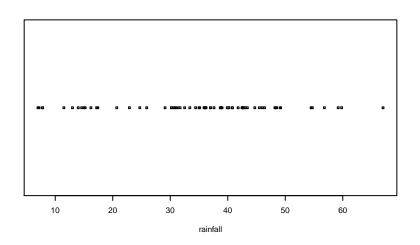


Figure: Stripchart of precip

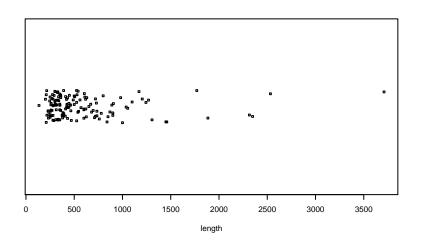


Figure: Stripchart of rivers



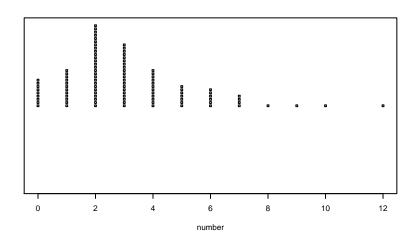


Figure: Stripchart of discoveries



Histograms

- Histograms
 - typically for continuous data
 - decide on bins/classes, make bars proportional to membership
 - often misidentified (bar graphs)
- > hist(precip, main = "")
- > hist(precip, freq = FALSE, main = "")

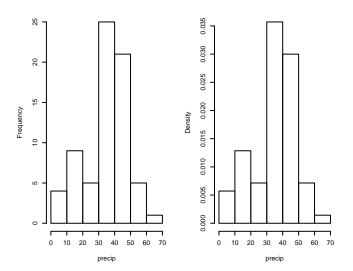


Figure: Histograms of precip

Remarks about histograms

- choose different bins, get a different histogram
- many algorithms for choosing bins automatically
- should investigate several bin choices
 - look for stability
 - try to capture underlying story of data

Stemplots

- Stemplots have two basic parts: stems and leaves
 - initial digit(s) taken for stem
 - trailing digits stand for leaves
 - leaves accumulate to the right

Example

Road Casualties in Great Britain 1969-84. A time series of total car drivers killed or seriously injured in Great Britain monthly from Jan 1969 to Dec 1984.

Stemplot of UK Driver Deaths

- > library(aplpack)
- > stem.leaf(UKDriverDeaths, depth = FALSE)

```
1 | 2: represents 120
leaf unit: 10
            n: 192
   10 | 57
   11 | 136678
   12 | 123889
   13 L 0255666888899
   14 | 00001222344444555556667788889
   15 | 0000111112222223444455555566677779
   16 | 01222333444445555555678888889
   17 | 11233344566667799
   18 | 00011235568
   19 | 01234455667799
   20 | 0000113557788899
   21 | 145599
   22 | 013467
   23 I 9
   24 I 7
HT: 2654
```

Code for stemplots

```
> UKDriverDeaths[1:4]
[1] 1687 1508 1507 1385
> stem.leaf(UKDriverDeaths, depth = FALSE)
1 | 2: represents 120
leaf unit: 10
             n: 192
   10 | 57
        136678
   12 | 123889
        0255666888899
        00001222344444555556667788889
         0000111112222223444455555566677779
     G. Jav Kerns, Youngstown State University Probability and Statistics
```

Index Plots

Good for plotting data ordered in time

- a 2-D plot, with index (observation number) on x-axis,
 value on y-axis
- two methods
 - spikes: draws vertical line up to value (type = "h")
 - points: simple dot at the observed height (type = "p")

Example

Level of Lake Huron 1875-1972. annual measurements of the level (in feet) of Lake Huron from 1875–1972.



Index Plots

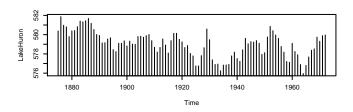
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- two methods
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Example

Level of Lake Huron 1875-1972. annual measurements of the level (in feet) of Lake Huron from 1875–1972.





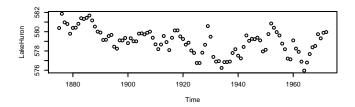


Figure: Index plots of LakeHuron

Qualitative Data, Categorical Data, Factors

- Qualitative data: any data that are not numerical, or do not represent numerical quantities
 - some data look qualitative. Example: shoe size
 - some data identify the observation, not of much interest
- Factors subdivide data into categories
 - possible values of a factor: levels
 - factors may be nominal or ordinal
 - nominal: levels are names, only (gender, political party, ethnicity)
 - ordinal: levels are ordered (SES, class rank, shoe size)

U.S. State Facts and Features. postal abbreviations

```
> str(state.abb)
chr [1:50] "AL" "AK" "AZ" "AR" ...
```

Example

U.S. State Facts and Features. The region in which a state resides

```
> state.region[1:4]
[1] South West West South
4 Levels: Northeast South ... West
```

Qualitative Data

- Factors have special status in R
 - represented internally by numbers, but not always printed that way
 - constructed with factor command
- Displaying Qualitative Data
 - first try: make a (contingency) table with table function
 - prop.table makes a relative frequency table

Example

U.S. State Facts and Features. State division



Displaying Qualitative Data

```
> Tbl <- table(state.division)</pre>
> Tb1
                     # frequencies
state.division
       New England Middle Atlantic
                  6
                                      3
    South Atlantic East South Central
                  8
West South Central East North Central
                                      5
West North Central
                              Mountain
                                      8
           Pacific
```

Displaying Qualitative Data

```
> Tbl/sum(Tbl)
                     # relative frequencies
state.division
                      Middle Atlantic
       New England
              0.12
                                  0.06
    South Atlantic East South Central
              0.16
                                  0.08
West South Central East North Central
              0.08
                                  0.10
West North Central
                              Mountain
              0.14
                                  0.16
           Pacific
              0.10
```

Displaying Qualitative Data

```
> prop.table(Tbl) # same thing
state.division
                      Middle Atlantic
       New England
              0.12
                                 0.06
    South Atlantic East South Central
              0.16
                                  0.08
West South Central East North Central
              0.08
                                  0.10
West North Central
                             Mountain
              0.14
                                 0.16
           Pacific
              0.10
```

Bar Graphs

- discrete analogue of the histogram
- make bar for each level of a factor
- may show frequencies or relative frequencies
- impression given depends on order of bars (default: alphabetical)

Example

U.S. State Facts and Features. State region

- > barplot(table(state.region))
- > barplot(prop.table(table(state.region)))



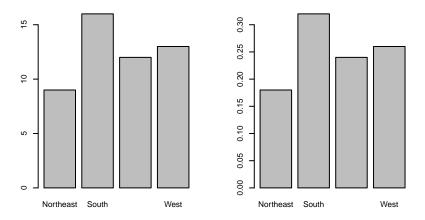


Figure: (Relative) frequency bar graphs of state.region

Pareto Diagram

- a bar graph with ordered bars
- bar with highest (relative) frequency goes on left
- bars drop from left to right
- can sometimes help discern hidden structure

Example

U.S. State Facts and Features. State division

- > library(qcc)
- > pareto.chart(table(state.division),
- + ylab = "Frequency")

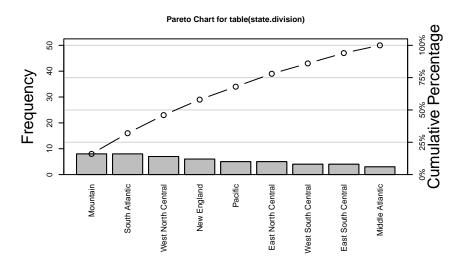


Figure: Pareto diagram of state.division



Dot Charts

- a bar graph on its side
- has dots instead of bars
- can show complicated multivariate relationships

Example

U.S. State Facts and Features. State region

- > x <- table(state.region)
- > dotchart(as.vector(x), labels = names(x))

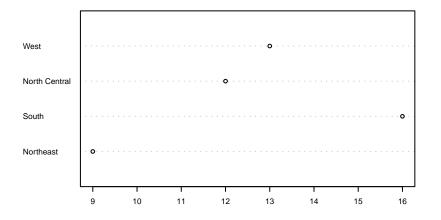


Figure: Dot chart of state.region



Other Data Types

Logical

> y

[1] TRUE TRUE TRUE FALSE FALSE

> !y

[1] FALSE FALSE TRUE TRUE

Missing



Other Data Types

Missing: represented by NA

>
$$x \leftarrow c(3, 7, NA, 4, 7)$$

> $y \leftarrow c(5, NA, 1, 2, 2)$
> $x + y$
[1] 8 NA NA 6 9

Some functions have na.rm argument

```
> is.na(x)
```

[1] FALSE FALSE TRUE FALSE FALSE

$$> z \leftarrow x[!is.na(x)]$$

[1] 21



Features of Data Distributions

Four Basic Features

- ① Center: middle or general tendency
- Spread: small means tightly clustered, large means highly variable
- Shape: symmetry versus skewness, kurtosis
- 4 Unusual Features: anything else that pops out at you about the data

More about shape

Symmetry versus Skewness

- symmetric
- right (positive) and left (negative) skewness

Kurtosis

- leptokurtic steep peak, heavy tails
- platykurtic flatter, thin tails
- mesokurtic right in the middle



Unusual features: clusters or gaps

> stem.leaf(faithful\$eruptions)

```
1 | 2: represents 1.2
leaf unit: 0.1
          n: 272
  12
        s | 667777777777
     2* | 0000000000011111111
       t | 22222222333333
        f | 44444
        s I 66
  94
       2. I 889
       3* I 0
 102
      t | 3333
 108
       f | 445555
 118
        s I 6666677777
 (16)
       3. | 888888889999999
 138
       4* | 00000000000000011111111111111
 107
        t | 22222222222333333333333333333
  78
            44444444444455555555555555555555555
  43
        s | 6666666666677777777777
  21
       4. | 888888888899999
       5* I 0001
```

Unusual features: extreme observations

- Extreme observation: falls far from the rest of the data
 - possible sources
 - could be typo
 - could be in wrong study
 - could be indicative of something deeper
- Quantitatively measure features: Descriptive Statistics
 - qualitative data: frequencies or relative frequencies
 - quantitative data: measures of CUSS

Measures of center: sample mean \overline{x} (read "x-bar"):

$$\overline{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{1}{n} \sum_{i=1}^{n} x_i.$$
 (1)

- Good: natural, easy to compute, nice properties
- Bad: sensitive to extreme values

- > stack.loss # built-in data
 - [1] 42 37 37 28 18 18 19 20 15 14 14 13 11
- [14] 12 8 7 8 8 9 15 15
- > mean(stack.loss)
- [1] 17.52381



Measures of center: sample median \tilde{x}

How to find it

- \bigcirc sort the data into an increasing sequence of n numbers
- ② \tilde{x} lies in position (n+1)/2
 - Good: resistant to extreme values, easy to describe
 - Bad: not as mathematically tractable, need to sort the data to calculate

How to do it with R

> median(stack.loss)

[1] 15



Measures of center: trimmed mean \overline{x}_t

How to find it

- "trim" a proportion of data from both ends of the ordered list
- a find the sample mean of what's left
 - Good: also resistant to extreme values, has good properties, too
 - Bad: still need to sort data to get rid of outliers

- > mean(stack.loss, trim = 0.05)
- [1] 16.78947



Order statistics

Given data x_1, x_2, \ldots, x_n , sort in an increasing sequence

$$x_{(1)} \le x_{(2)} \le x_{(3)} \le \cdots \le x_{(n)}$$
 (2)

- $x_{(k)}$ is the k^{th} order statistic
- approx 100(k/n)% of the observations fall below $x_{(k)}$

How to do it with R

> sort(stack.loss)

[1] 7 8 8 8 9 11 12 13 14 14 15 15 15

[14] 18 18 19 20 28 37 37 42



Sample quantile, order p (0 $\leq p \leq$ 1), denoted \tilde{q}_p

We describe the default (type = 7)

- ① get the order statistics $x_{(1)}, x_{(2)}, \ldots, x_{(n)}$.
- ② calculate (n-1)p+1, write in form k.d, with k an integer and d a decimal

3

$$\tilde{q}_p = x_{(k)} + d(x_{(k+1)} - x_{(k)}).$$
 (3)

ullet approximately 100p% of the data fall below the value $ilde{q}_p$.

How to do it with R

> quantile(stack.loss, probs = c(0, 0.25, 0.37))

0% 25% 37%

7.0 11.0 13.4

Measures of spread: sample variance, std. deviation

The sample variance s^2

$$s^{2} = \frac{1}{n-1} \sum_{i=1}^{n} (x_{i} - \overline{x})^{2}$$
 (4)

The sample standard deviation is $s = \sqrt{s^2}$.

- Good: tractable, nice mathematical/statistical properties
- Bad: sensitive to extreme values

- > var(stack.loss); sd(stack.loss)
- [1] 103.4619
- [1] 10.17162

Interpretation of s

Chebychev's Rule:

The proportion of observations within k standard deviations of the mean is at least $1 - 1/k^2$, *i.e.*, at least 75%, 89%, and 94% of the data are within 2, 3, and 4 standard deviations of the mean, respectively.

Empirical Rule:

If data follow a bell-shaped curve, then approximately 68%, 95%, and 99.7% of the data are within 1, 2, and 3 standard deviations of the mean, respectively.

Measures of spread: interquartile range

The Interquartile range IQR

$$IQR = \tilde{q}_{0.75} - \tilde{q}_{0.25} \tag{5}$$

- Good: resistant to outliers
- Bad: only considers middle 50% of the data

How to do it with R

> IQR(stack.loss)

[1] 8



Measures of spread: median absolute deviation

The median absolute deviation MAD:

- ① get the order statistics, find the median \tilde{x} .
- 2 calculate the absolute deviations:

$$|x_1-\tilde{x}|, |x_2-\tilde{x}|, \ldots, |x_n-\tilde{x}|$$

- 3 the $MAD \propto \text{median} \{ |x_1 \tilde{x}|, |x_2 \tilde{x}|, \dots, |x_n \tilde{x}| \}$
 - Good: excellently robust
 - Bad: not as popular, not as intuitive

- > mad(stack.loss)
- Γ1] 5.9304



Measures of spread: the range

The range R:

$$R = x_{(n)} - x_{(1)} \tag{6}$$

- Good (not so much): easy to describe and calculate
- Bad: ignores everything but the most extreme observations

```
> range(stack.loss)
```

- [1] 7 42
- > diff(range(stack.loss))
- [1] 35



Measures of shape: sample skewness

The sample skewness g_1 :

$$g_1 = \frac{1}{n} \frac{\sum_{i=1}^{n} (x_i - \overline{x})^3}{s^3}.$$
 (7)

Things to notice:

- invariant w.r.t. location and scale
- \bullet $-\infty < g_1 < \infty$
- sign of g_1 indicates direction of skewness (\pm)

- > library(e1071)
- > skewness(stack.loss)
- [1] 1.156401



Measures of shape: sample skewness

How big is BIG?

Rule of thumb:

If $|g_1| > 2\sqrt{6/n}$, then the data distribution is substantially skewed (in the direction of the sign of g_1).

- > skewness(discoveries)
- [1] 1.207600
- > 2 * sqrt(6/length(discoveries))
- [1] 0.4898979



Measures of shape: sample excess kurtosis

The sample excess kurtosis g_2 :

$$g_2 = \frac{1}{n} \frac{\sum_{i=1}^{n} (x_i - \overline{x})^4}{s^4} - 3.$$
 (9)

Things to note:

- invariant w.r.t. location and scale
- $-2 ≤ g_2 < ∞$
- $g_2 > 0$ indicates leptokurtosis, $g_2 < 0$ indicates platykurtosis

- > library(e1071)
- > kurtosis(stack.loss)
- [1] 0.1343524

Measures of shape: sample excess kurtosis

Again, how big is BIG?

Rule of thumb:

If $|g_2| > 4\sqrt{6/n}$, then the data distribution is substantially kurtic.

- > kurtosis(UKDriverDeaths)
- [1] 0.07133848
- > 4 * sqrt(6/length(UKDriverDeaths))
- [1] 0.7071068



Exploratory data analysis: more on stemplots

- Trim Outliers: observations that fall far from the bulk of the other data often obscure structure to the data and are best left out. Use the trim.outliers argument to stem.leaf.
- Split Stems: we sometimes fix "skyscraper" stemplots by increasing the number of lines available for a given stem.
 The end result is a more spread out stemplot which often looks better. Use the m argument to stem.leaf
- Depths: give insight into balance of the data around the median. Frequencies are accumulated from the outside inward, including outliers. Use depths = TRUE.

More about stemplots

> stem.leaf(faithful\$eruptions)

```
1 | 2: represents 1.2
leaf unit: 0.1
          n: 272
  12
        s | 667777777777
     2* | 0000000000011111111
        t | 22222222333333
        f | 44444
        s I 66
  94
       2. I 889
       3* I 0
 102
       t | 3333
 108
        f | 445555
 118
        s I 6666677777
 (16)
       3. I 888888889999999
 138
       4* | 00000000000000011111111111111
 107
        t | 22222222222333333333333333333
  78
            44444444444455555555555555555555555
  43
        s | 6666666666677777777777
  21
       4. | 888888888899999
       5* I 0001
```

Hinges and the 5*NS*

- Find the order statistics $x_{(1)}, x_{(2)}, \ldots, x_{(n)}$.
- The *lower hinge* h_L is in position $L = \lfloor (n+3)/2 \rfloor /2$
- The *upper hinge* h_U is in position n + 1 L.

Given the hinges, the five number summary (5NS) is

$$5NS = (x_{(1)}, h_L, \tilde{x}, h_U, x_{(n)}). \tag{10}$$

- > fivenum(stack.loss)
- [1] 7 11 15 19 42



Boxplots

Boxplot: a visual display of the 5*NS*. Can visually assess multiple features of the data set:

- Center: estimated by the sample median, \tilde{x}
- *Spread:* judged by the width of the box, $h_U h_L$
- Shape: indicated by the relative lengths of the whiskers, position of the median inside box.
- Extreme observations: identified by open circles

How to do it with R

> boxplot(rivers, horizontal = TRUE)



Outliers

potential: falls beyond 1.5 times the width of the box

less than
$$h_L - 1.5(h_U - h_L)$$
 or greater than $h_U + 1.5(h_U - h_L)$

suspected: falls beyond 3 times the width of the box

less than
$$h_L - 3(h_U - h_L)$$
 or greater than $h_U + 3(h_U - h_L)$

- > boxplot.stats(rivers)\$out
 - [1] 1459 1450 1243 2348 3710 2315 2533 1306
 - [9] 1270 1885 1770



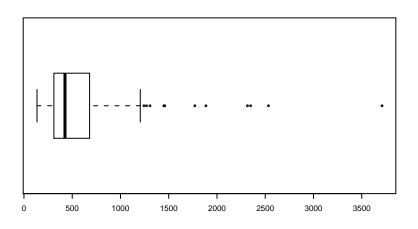


Figure: Boxplot of rivers

Standardizing variables

- useful to see how observation relates to other observations
- AKA measure of relative standing, z-score

$$z_i = \frac{x_i - \overline{x}}{s}, \quad i = 1, 2, \dots, n$$

- unitless
- positive (negative) z-score falls above (below) mean

- > scale(precip)[1:3]
- [1] 2.342971 1.445597 -2.034466



Multivariate data: data frames

- usually have two (or more) measurements associated with each subject
- display in rectangular array
 - each row corresponds to a subject
 - columns contain the measurements for each variable

- > x <- 5:6; y <- letters[3:4]; z <- c(0.1, 3.8)
 > data.frame(v1 = x, v2 = y, v3 = z)
 v1 v2 v3
 1 5 c 0.1
- 2 6 d 3.8

More on data frames

- must have same number of rows in each column
- all measurements in single column must be same type
- indexing is two-dimensional; the columns have names

```
> A <- data.frame(v1 = x, v2 = y, v3 = z)
> A[2, 1]; A[1,]; A[, 3]
[1] 6
   v1 v2 v3
1 5 c 0.1
[1] 0.1 3.8
```

Bivariate data: qualitative versus qualitative

Two categorical variables

- usually make a two-way contingency table
- in the R Commander with Statistics ▷ Contingency Tables

```
> library(RcmdrPlugin.IPSUR)
```

```
> data(RcmdrTestDrive)
```

```
> xtabs(~ gender + smoking, data = RcmdrTestDrive)
        smoking
```

```
Nonsmoker Smoker
gender
  Female
                 61
  Male
                 75
                         23
```

Bivariate data: more on tables

- Descriptive statistics: for now, marginal totals/percentages
- more to talk about later: odds ratio, relative risk

How to do it with R

```
> A <- xtabs(Freq ~ Survived + Class, data = Titanic)
> addmargins(A)
```

Class

```
    Survived
    1st
    2nd
    3rd
    Crew
    Sum

    No
    122
    167
    528
    673
    1490

    Yes
    203
    118
    178
    212
    711

    Sum
    325
    285
    706
    885
    2201
```



Bivariate data: more on tables

```
> library(abind)
```

> colPercents(A)

Class

```
    Survived
    1st
    2nd
    3rd
    Crew

    No
    37.5
    58.6
    74.8
    76

    Yes
    62.5
    41.4
    25.2
    24

    Total
    100.0
    100.0
    100.0
    100

    Count
    325.0
    285.0
    706.0
    885
```

> rowPercents(A)

Class

Survived 1st 2nd 3rd Crew Total Count

No 8.2 11.2 35.4 45.2 100 1490

Yes 28.6 16.6 25.0 29.8 100 711

Plotting two categorical variables

- Stacked bar charts
- Side-by-side bar charts
- Spine plots

- > barplot(A, legend.text = TRUE)
- > barplot(A, legend.text = TRUE, beside = TRUE)
- > spineplot(A)

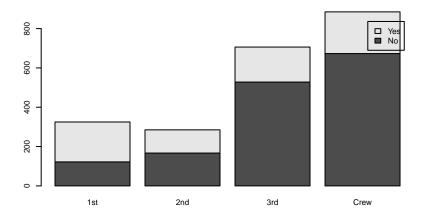


Figure: Stacked bar chart of Titanic data



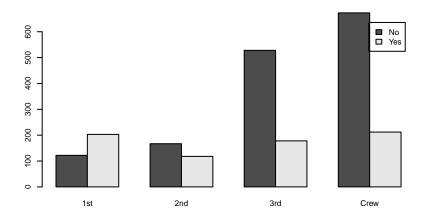


Figure: Side-by-side bar chart of Titanic data



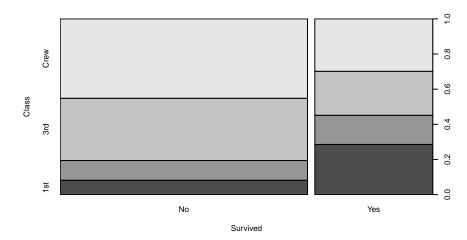


Figure: Spine plot of Titanic data

Bivariate data: quantitative versus quantitative

- Can do univariate graphs of both variables separately
- Make scatterplots for both variables simultaneously

- > plot(conc ~ rate, data = Puromycin)
- > library(lattice)
- > xyplot(conc ~ rate, data = Puromycin)

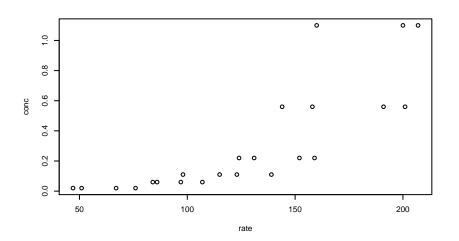


Figure: Scatterplot of Puromycin data



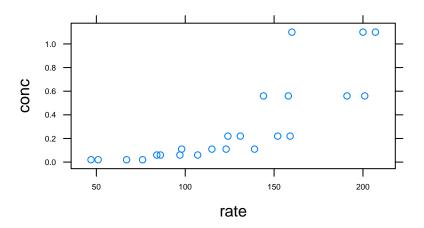


Figure: Scatterplot of Puromycin data



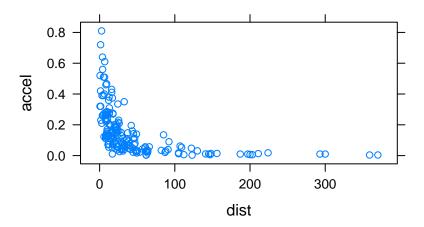


Figure: Scatterplot of attenu data



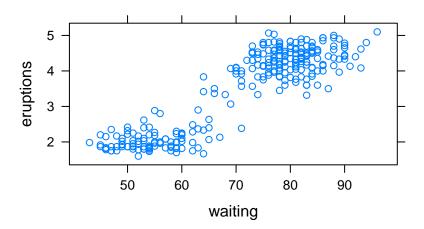


Figure: Scatterplot of faithful data

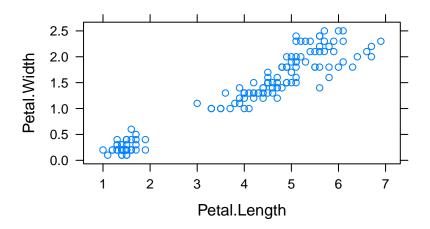


Figure: Scatterplot of iris data

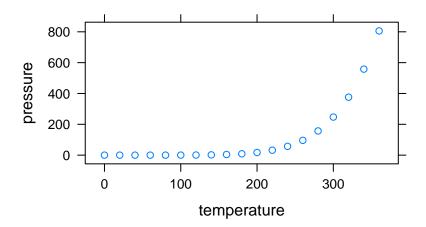


Figure: Scatterplot of iris data

Measuring Linear association

The sample Pearson product-moment correlation coefficient:

$$r = \frac{\sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \overline{x})} \sqrt{\sum_{i=1}^{n} (y_i - \overline{y})}}$$

- independent of scale
- $-1 \le r \le 1$, equality when points lie on straight line

How to do it with R

- > with(iris, cor(Petal.Width, Petal.Length))
- [1] 0.9628654
- > with(attenu, cor(dist, accel))
- [1] -0.4713809

More about linear correlation

- measures strength and direction of linear association
- Rules of thumb:
 - 0 < |r| < 0.3, weak linear association
 - 0.3 < |r| < 0.7, moderate linear association
 - ullet 0.7 < |r| < 1, strong linear association
- Just because $r \approx 0$ doesn't mean there isn't any association

One quantitative, one categorical

- Break down quantitative var by groups of subjects
 - compare centers and spreads: variation within versus between groups
 - compare clusters and gaps
 - compare outliers and unusual features
 - compare shapes.
- graphical and numerical

Comparison of groups

How to do it with R

```
> stripchart(weight ~ feed, method = "stack",
+     data = chickwts)
> library(lattice)
> histogram(~age | education, data = infert)
> bwplot(~count | spray, data = InsectSprays)
```

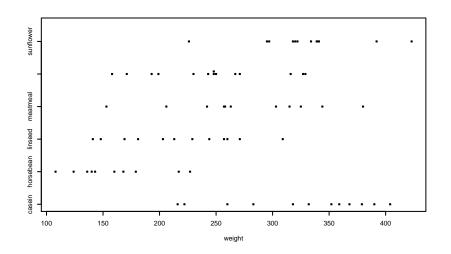


Figure: Stripcharts of chickwts data



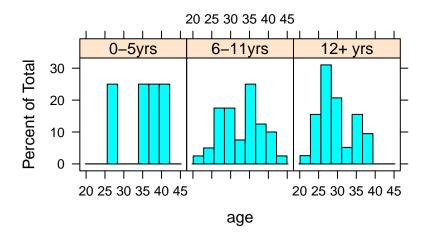


Figure: Histograms of infert data



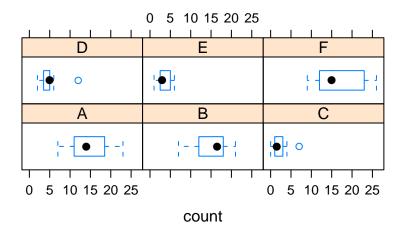


Figure: Boxplots of InsectSprays data

Multiple variables

With more variables, complexity increases

- multi-way contingency tables (bunch of categorical vars)
 - mosaic plots, dotcharts
- sample variance-covariance matrices
 - scatterplot matrices
- comparing groups: coplots

How to do it with R

```
> splom(~cbind(Murder, Assault, Rape),
+ data = USArrests)
> `?`(dotchart)
> `?`(xyplot)
> `?`(mosaicplot)
```

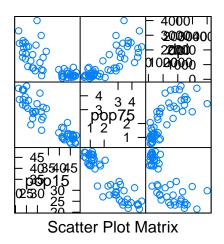


Figure: Scatterplot matrix of LifeCycleSavings data

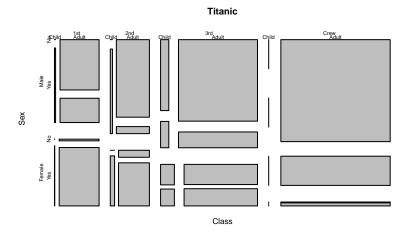
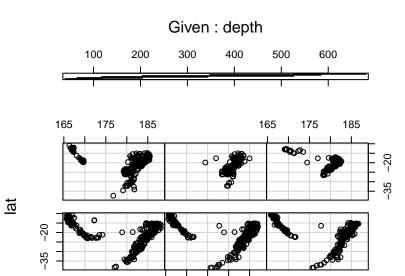


Figure: Mosaic plot of Titanic data



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