Continuous R.V.s

X is continuous if F (CDF) is differentiable at almost all x and

$$\int_{-\infty}^{\infty} F'(x)\,dx = 1.$$

Discussion:

1) Unlike discrete r.v.'s

$$h(x) = \mathbb{P}(X = x) = 0.$$

for all x in continuous case

2) We let

$$F(x) = \mathbb{P}(X \le x) = \int_{-\infty}^{x} F'(u)\,du$$

$\Rightarrow$ F is continuous.

Def^n: The PROBABILITY DENSITY FUNCTION (PDF) is

$$f(x) = f_X(x) = F'(x)$$

COMPARISON

DISCRETE	CONTINUOUS
$p(x)$ PMF	$f(x)$ PDF
$p(x) \geq 0$	$f(x) \geq 0$
$\sum_x p(x) = 1$	$\int_{-\infty}^{\infty} f(x)\,dx = 1$
$\mathbb{P}(X \in A) = \sum_{x \in A} p(x)$	$\mathbb{P}(X \in A) = \int_A f(x)\,dx$
	$\mathbb{P}(a < X < b) = \int_a^b f(x)\,dx$

Example Have a PDF

$$f(x) = \begin{cases} Ce^{-2x}, & x \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

(for some constant C)

1) What is C?

KNOW:

$$1 = \int_{-\infty}^{\infty} f(x)\,dx$$

$$1 = \int_0^{\infty} Ce^{-2x}\,dx$$

$$1 = C\int_0^{\infty} e^{-2x}\,dx$$

$$1 = C\lim_{t\to\infty}\int_0^{t} e^{-2x}\,dx$$

$$1 = C\lim_{t\to\infty}\left(-\frac{1}{2}e^{-2x}\Big|_{x=0}^{t}\right)$$

$$1 = C\cdot\lim_{t\to\infty}\left(\frac{1}{2} - \frac{1}{2}e^{-2t}\right)$$

$$1 = C\cdot\frac{1}{2}$$

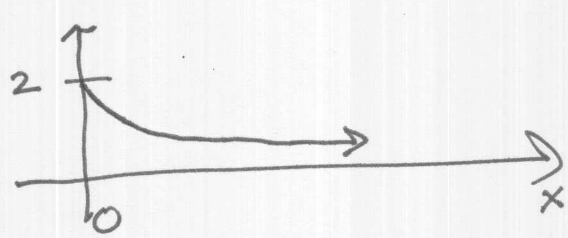
$$\boxed{\therefore\ C = 2}$$

In general,

$$\int_0^{\infty} \lambda e^{-\lambda x}\,dx = 1 \quad \text{for any } \lambda$$

2) $f(x) = 2e^{-2x}$, $x \geq 0$

DRAW GRAPH



3) $\mathbb{P}(1 \le X \le 3)$

$$= \int_1^3 f(x)\,dx$$

$$= \int_1^3 2e^{-2x}\,dx$$

$$= 2\int_1^3 e^{-2x}\,dx$$

$$= \cancel{2} \cdot -\frac{1}{\cancel{2}} e^{-2x} \Big|_{x=1}^{3}$$

$$= e^{-2(1)} - e^{-2(3)}$$

$$= e^{-2} - e^{-6} \approx \text{decimal}$$

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4) Find CDF

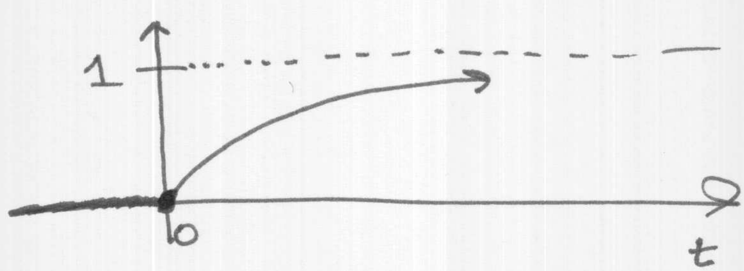
$$F(t) = \mathbb{P}(X \le t)$$

$$= \begin{cases} 0 & , \text{ if } t < 0 \\ \int_0^t f(x)\,dx & , \text{ if } t \ge 0 \end{cases}$$

$$\int_0^t 2e^{-2x}\,dx = -e^{-2x}\Big|_{x=0}^{t}$$

$$= 1 - e^{-2t}, \; t \ge 0$$

DRAW GRAPH:



In general, for PDF $f(x) = \lambda e^{-\lambda x}$ the CDF is

$$\boxed{F(x) = 1 - e^{-\lambda x}, \; x > 0}$$

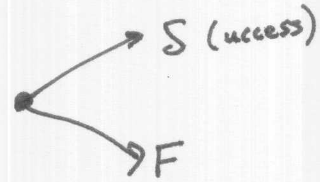
Remark: λ is a parameter.

$f(x) = \lambda e^{-\lambda x}, x > 0$ for $\lambda > 0$ is a PARAMETRIC FAMILY

FAMOUS DISCRETE MODELS

BINOMIAL MODEL

"Bernoulli trial"



$\mathbb{P}(S) = p$

$\mathbb{P}(F) = 1 - p$

Based on 3 assumptions

1) conduct n Bernoulli trials
2) trials are independent
3) prob of success p is constant

If assumptions hold,
let X = # of successes

then
$$p(x) = \mathbb{P}(X = x) = \binom{n}{x} p^{x} (1-p)^{n-x}, \quad x = 0, 1, 2, \ldots, n.$$