9/10/10

Ex] Car Wash

On avg. 7 cars arrive every hour. Let

$\underline{X}$ = # cars arrive from 10AM - 11AM

$\underline{X} \sim pois(\lambda = 7)$

$\mathbb{P}$(no car arrives)

$= \mathbb{P}(X = 0)$

$= e^{-\lambda} \frac{\lambda^0}{0!} = e^{-7}$

$\mathbb{P}$(more than 4 cars)

$= \mathbb{P}(X > 4)$

$= \mathbb{P}(X \geq 5)$

$= 1 - \mathbb{P}(0 \leq X \leq 4)$

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Ex] Let $Y$ be the number of cars from 8AM to 3PM.

Then $Y \sim pois(7 \times 7 = 49)$

$\mathbb{P}$(between 48 and 52 cars, inclusive)

$= \mathbb{P}(48 \leq X \leq 52)$

$= \mathbb{P}(X = 48$ or $X = 49$ or ... or $X = 52)$

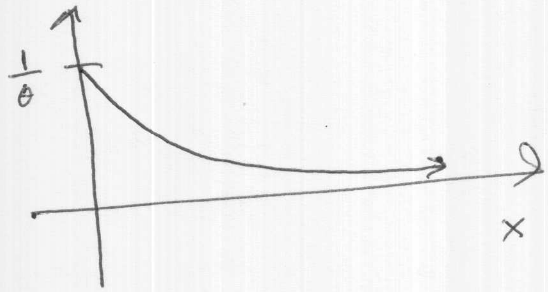
$= \mathbb{P}(X = 48) + \cdots + \mathbb{P}(X = 52)$

$= \sum_{x=48}^{52} e^{-49} \cdot \frac{49^x}{x!}$

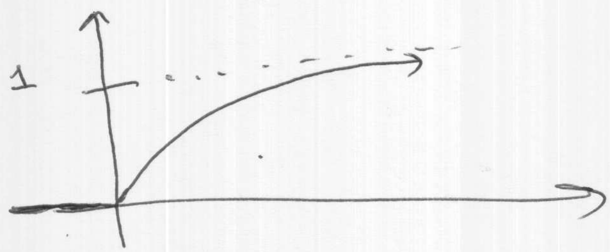
# Exponential Model

$X \sim \exp(\theta)$ if the PDF is

$$f(x) = \frac{1}{\theta} e^{-x/\theta}, \quad x > 0$$




CDF: $F(x) = 1 - e^{-x/\theta}, \quad x > 0$




Check: $f(x) \geq 0$, $\int_{-\infty}^{\infty} f(x)\,dx = 1$ } already done

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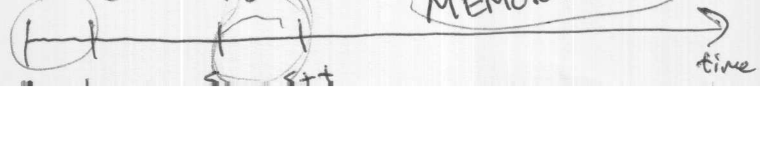
Exponential distribution assoc. with waiting times.

For exponential dist'ns:

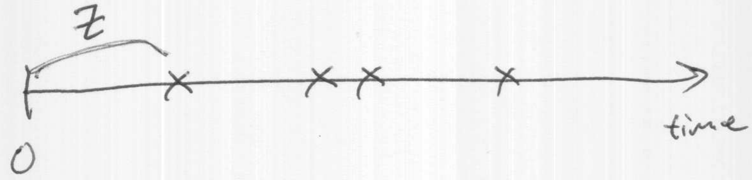
$$\mathbb{P}(X > s+t \mid X > s) = \mathbb{P}(X > t)$$

MEMORYLESS



# Relationship to Poisson Model

$\lambda$ = rate of occurrences of "rare events"




Let $Y$ = # events in $[0, t)$.

Then know that

$$Y \sim \text{pois}(\lambda t)$$

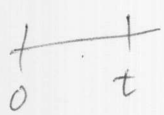
and the time until $1^{st}$ event occurs is $Z \sim \exp(1/\lambda)$

## Why?:

Look at CDF of $Z$.

$$F_Z(t) = \mathbb{P}(Z \le t)$$

$$= 1 - \mathbb{P}(Z > t)$$




$$= 1 - \mathbb{P}(\text{no events in } [0,t))$$

$$= 1 - \mathbb{P}(Y = 0)$$

$$= 1 - e^{-\lambda t}$$

Then the PDF of $Z$ is

$$F_Z'(t) = -e^{-\lambda t} \cdot (-\lambda)$$

$$= \lambda e^{-\lambda t}, \quad t > 0$$

$$= \frac{1}{(1/\lambda)} e^{-t/(1/\lambda)}, \quad t > 0$$

So $Z \sim \exp(1/\lambda)$

# GAUSSIAN MODEL

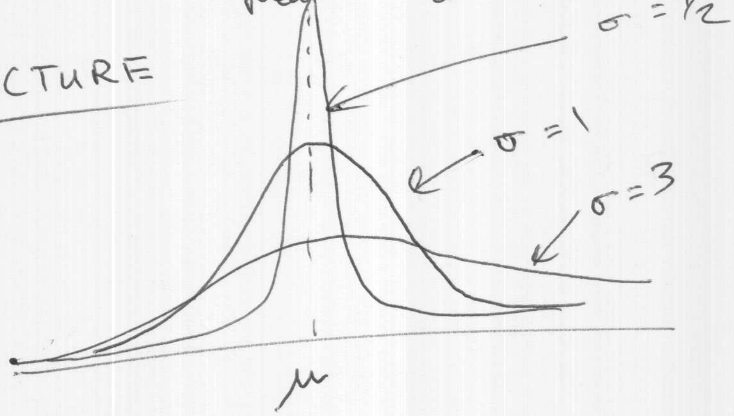
AKA Normal Dist'n

AKA Bell Curve

$$X \sim \text{norm}(\mu, \sigma)$$

mean — standard deviation

## PICTURE




## PDF

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty$$

$$\sigma > 0$$

## Check:

① $f(x) \geq 0$

② $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\left( \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx \right)^2 = 1^2$$