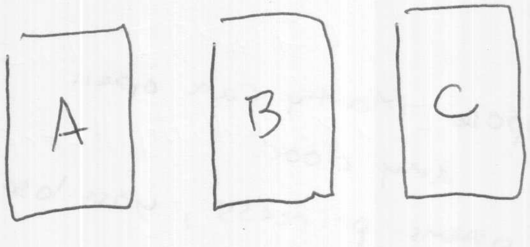
Monty Hall

9/22/10 (1)



Pick door: A.

Monty opens door. (Say C)

Allowed to switch.

What's better? switch or stay?

Let $A = \{$princess behind door A$\}$

$B = \{$ " " " B $\}$

$C = \{$ " " " C $\}$

Let $M = \{$Monty opens door C$\}$.

Would like to know:

$$\mathbb{P}(A|M) = \frac{\mathbb{P}(A\cap M)}{\mathbb{P}(M)}$$

$$\mathbb{P}(M) = \mathbb{P}(M\cap S)$$

$$= \mathbb{P}(M\cap\{A\cup B\cup C\})$$

$$= \mathbb{P}(M\cap A) + \mathbb{P}(M\cap B) + \mathbb{P}(M\cap C)$$

$$= \mathbb{P}(A)\mathbb{P}(M|A) + \mathbb{P}(B)\mathbb{P}(M|B) + \cancel{\mathbb{P}(C)\mathbb{P}(M|C)}$$

$$= \frac{1}{3}\cdot\frac{1}{2} + \frac{1}{3}\cdot 1 + \cancel{\frac{1}{3}\cdot 0}$$

$$\mathbb{P}(M) = \frac{1}{3}\cdot\frac{1}{2} + \frac{1}{3}$$

$$\mathbb{P}(A|M) = \frac{1/3\cdot\frac{1}{2}}{\frac{1}{3}\cdot\frac{1}{2} + \frac{1}{3}} = \frac{\frac{1}{6}}{3/6} = \frac{1}{3}$$

$$\mathbb{P}(B|M) = \frac{\frac{1}{3}}{\frac{1}{3}\cdot\frac{1}{2}\cdot\frac{1}{3}} = \frac{\frac{2}{6}}{3/6} = \frac{2}{3}$$

9/22/10 (2)

Expectation

Describe some characteristics of a R.V.

Average value of X?

Central value ?

Long term behavior?

Intuition: If X is discrete.

x	a	b
$f(x)$	p	$1-p$

Repeat experiment over and over.

Calculate

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

$$= a \cdot \frac{\#(a)}{n} + b \frac{\#(b)}{n}$$

$$\xrightarrow[n \to \infty]{\text{limit}} a \cdot p + b(1-p)$$

Expectation is the limit

$$\lim_{n \to \infty} \bar{x} = ap + (1-p)b.$$

Def^n: If X (discrete) has PMF f_X then

$$\mathbb{E}X = \sum_{i=1}^{\infty} x_i f(x_i)$$

$$= x_1 f(x_1) + x_2 f(x_2) + \cdots$$

(Continuous) X has PDF $f(x)$

$$\mathbb{E}(X) \approx \sum x f(x)$$

$$\boxed{\mathbb{E}X = \int_{-\infty}^{\infty} x f(x)\,dx}$$

In general, if $Y = g(X)$

Then

$$\mathbb{E}Y = \mathbb{E}g(X) = \int_{-\infty}^{\infty} g(x) f(x)\,dx$$

<u>Note</u>: all of this is provided that $\mathbb{E}|g(X)| < \infty$

i.e. $\sum_{-\infty}^{\infty} |g(x)| f(x) < \infty$ or

$$\int_{-\infty}^{\infty} |g(x)| f(x)\,dx < \infty.$$

if this doesn't happen, then

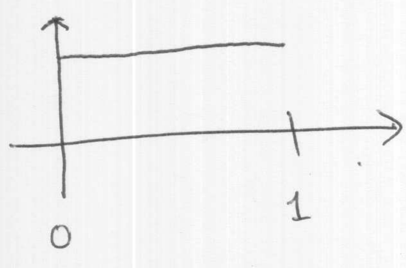
$\mathbb{E}g(X)$ <u>does</u> <u>not</u> <u>exist</u>.

Ex) Bernoulli $\quad X =$ # heads in 1 trial

x	0	1
$f(x)$	$1-p$	p

$$\mathbb{E}X = 0\cdot(1-p) + 1(p) = p.$$

Ex) <u>Uniform</u>: $f(x) = 1,\ 0<x<1.$



$\mathbb{E}X = 1/2$

$\int_0^1 x \cdot 1\,dx$

$= \frac{x^2}{2}\Big|_{x=0}^{1} = 1/2$

<u>More is true</u>: If $f(x)$ is symmetric about the point $x=\mu$ then

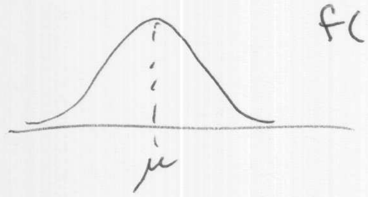
$$\mathbb{E}X = \mu \text{ (when it exists)}$$

Ex) Normal Dist'n. 9/22/10 (4)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty$$

$\mathbb{E}X = \mu$ Since f is symmetric about $x = \mu$

$f(x) = h(|x-\mu|)$



Properties for any a, b and any X, Y with $\mathbb{E}|X| < \infty$ and $\mathbb{E}|Y| < \infty$

a) $\mathbb{E}\, aX = a\,\mathbb{E}X$

b) If g_1 and g_2 are any 2 functions with $\mathbb{E}|g_1(X)| < \infty$ and $\mathbb{E}|g_2(X)| < \infty$

then $\mathbb{E}\left[g_1(X) + g_2(X)\right]$

$= \mathbb{E}g_1(X) + \mathbb{E}g_2(X)$

Properties a) + b) $\Rightarrow$ $\mathbb{E}$ is a "linear operator"

Higher Moments of X:

$\mathbb{E}X^k = k^{th}$ moment of X about zero

$\mathbb{E}(X-b)^k = k^{th}$ moment of X about b.

Variance $\mathbb{E}(X-\mu)^2 \Rightarrow$ "2^{nd} central moment"

$\sigma^2 = \mathbb{E}(X-\mu)^2$

$\sigma = \sqrt{\sigma^2}$ is STANDARD DEVIATION

Properties of Variance

1) $\sigma^2 \geq 0$

2) If $Y = aX + b$

Then $\sigma_Y^2 = a^2 \sigma_X^2$

$\sigma_Y = |a| \cdot \sigma_X$

Examples: Bernoulli

x	0	1
$f(x)$	$1-p$	p

$\mathbb{E}X = p$

$$\mathbb{E}(X-p)^2 = (0-p)^2 \cdot (1-p) + (1-p)^2 \cdot p$$

$$= p^2(1-p) + (1-p)^2 p$$

$$= (1-p)p(p + 1 - p)$$

$$\boxed{\sigma^2 = p(1-p)}$$

Shortcut:

$$\mathbb{E}(X-\mu)^2 = \mathbb{E}(X^2 - 2\mu X + \mu^2)$$

$$= \mathbb{E}X^2 - \mathbb{E}(2\mu X) + \mathbb{E}\mu^2$$

$$= \mathbb{E}X^2 - 2\mu \mathbb{E}X + \mathbb{E}\mu^2$$

$$= \mathbb{E}X - 2\mu^2 + \mu^2$$