9:00-10:00: Registration-Atrium
10:00-10:30: Breakfast and Introduction-Atrium

|  | Room 1106 | Room 1111 | Room 1120 | Room 1062: <br> Mathematical Biology | Room 2057 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 10:30-10:45 | David Freund | Michael Donzella | Jared Ruiz | Michaelangelo Tabone | Scott Eddy <br> $10: 50-11: 05$ |
| William Clemson | Jacob Shreffler | Erica Cross | Marisa Goldrich | Matt Grimm |  |
| $11: 10-11: 25$ | Paul Havens | Samantha Corvino | Todd Tichenor | Lisa Curll | John Hoffman |
| $11: 30-11: 45$ | Katie Burgoon | Sarah Ritchey | Doug Wajda | Christian Woods | Dean Zeller |
| $11: 50-12: 05$ | David Zach | Jeana Stevens | Neil Sandberg |  |  |

2009 Conference Schedule

|  | Room 1106 | Room 1111 | Room 1120 | Room 1062: <br> Mathematical Biology | Room 2057: COMAP |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1:00-1:15 | Bob Short | Jeff Burdette | Andrew Hosie | Vikram Raghu | MCM-A (YSU) |
| 1:20-1:35 | Kaylee Sutton | Krista Foster | Moriah Wright | Christopher Horvat | MCM-A (SRU) |
| 1:40-1:55 | Casey Dykes \& Anna Levina | Chim Chune Ko \& Aron Siegal | Dylan Sorge | Thomas Reid | MCM-A (YSU) |
| 2:00-2:15 | John-Tyler Soltys \& Richard Ligo | Emily Cunningham \& Josh Fitzgerald | Andrew Johnson | Kristen Pueschel | MCM-A (SRU) |
| 2:20-2:35 | Benjamin Mackey | W. Ryan Livingston | Michelle Cordier |  | MCM-B (YSU) |

2:40: Closing Remarks-Atrium

We all know the limit definition for the first derivative. Is there one for the second? The third? We'll see!

Room 1111

Maps that Preserve the Zeroes of Lie Polynomials of Degree 3<br>Kent State University<br>Advised by: Dr. Mikhail Chebotar

We present a brief introduction to linear preserver problems and Lie polynomials and give proofs for functions that preserve the zeroes of Lie polynomials of degree 3 over the field of complex matrices.

Jared Ruiz<br>The Beach Ball Problem<br>Youngstown State University<br>Advised by: Dr. Angela Spalsbury

Room 1120

For decades, festival and concert goers have been entertained by that one sacred object: the beach ball. For the duration of nearly every rock concert, most in the crowd seek out the hallowed beach ball, for no other reason than to smack it into the air. It would seem that many in the crowd have the opportunity to hit the beach ball quite a number of times, while others unfortunately never touch it at all. This presentation simulates a concert scenario, to finally answer who touches the ball the most, and give some advice for future concerts.

10:30-10:45<br>Michaelangelo Tabone<br>Ideal Pheromone Communication in Ant Foraging<br>University of Pittsburgh<br>Advised by: Dr. Brent Doiron

Room 1062

Ant colonies that forage in dynamic environments must shift attention from one spatial location to another as resources redistribute in time. This means that established pheromone trails leading to vanished food sources need to be removed, and new trails leading to more plentiful food need to be established. The pheromone evaporation rate constant determines both the ability of an ant colony to forget old trails as well as establish new trails. Through the combination of a simplified two trail motivated boundary value problem, and a full two dimensional self biasing random walk model, we show that there exists an ideal pheromone evaporation time constant which allows for the maximum utilization of food sources which change dynamically in space.

## 10:30-10:45

## Scott Eddy

Room 2057
On the Wave Equation
Youngstown State University
Advised by: Dr. George Yates

I will talk about the solution to the wave equation, some of its relations to an actual string, and an interesting discovery with regard to the wave equation's general solution.

What happens if you repeatedly take the derivative of a product of two functions? We'll find out!

10:50-11:05

Jacob Shreffler<br>An Algebraic Approach to Infinitesimal Analysis<br>Kent State University<br>Advised by: Dr. Joseph Diestel

Room 1111

Engineers, scientists, and many pre-19th century mathematicians have done a lot of work using the concept of infinitely small numbers. However, a logically rigorous foundation for infinitesimal analysis was not found until the 20th century. This talk is a brief introduction to Edwin Hewitt's method for algebraically generating the infinitesimal containing hyperreal number system by using a ring of continuous functions and without relying on advanced concepts from logic or non-classical set theory. Unlike other popular methods, Hewitt's method requires no nonstandard models, superstructures, or internal set theory axioms.

10:50-11:05<br>Erica Cross<br>Correlation Distribution in the Presence of Censoring<br>Youngstown State University<br>Advised by: Dr. Scott Linder

Room 1120

In many real world applications, experimental conditions impose censoring on data. Censoring imposes a dependence structure on the data which renders mathematically intractable sampling distributions of many commonly used statistics. We examine the sampling distribution of the sample correlation coeefficent and of Kendall's Tau when data arising from a bivariate normal model has been subjected to Type II censoring. We propose a simple approximation to this distribution and examine its goodness of fit.

Marissa Goldrich<br>Immune Response to Influenza Infection<br>University of Pittsburgh<br>Advised by: Dr. David Swigon

Influenza A is a contagious acute respiratory disease caused by a virus that infects epithelial cells in the respiratory tract. The virus is suppressed by the immune system of the host which consists of multiple components that act at variable stages of the virus reproduction. Using a published mathematical model of the host immune responses to the Influenza A virus, I will discuss the benefits and shortcomings of each type of immune response, based on the variation of the reproductive capacity of the virus.

10:50-11:05

## Matt Grimm

Room 2057

> A Characterization of Eigenvalues in Skew-Symmetric Matrices
> Kent State University
> Advised by: Dr. Stephen Gagola

I will look at the structure of eigenvalues in Skew-Symmetric matrices. We will then consider what conditions in these matrices satisfy certain properties of these eigenvalues.

This talk is a brief excursion into Fractional Calculus, focusing on the derivation of a definition for a Fractional Derivative using the Gamma function.

## 11:10-11:25

Samantha Corvino<br>Special Sums of Integer Reciprocals<br>Slippery Rock University<br>Advised by: Dr. Richard Marchand

Room 1111

A recent issue of Pentagon, the Kappa Mu Epsilon Journal, posed a problem requiring the sum of the reciprocals of the nonzero digits in a number formed by concatenating the integers from 1 to 999,999. The solution to this problem and its natural extension will be presented.

11:10-11:25
Todd Tichenor
Room 1120

## A Look at the RSA Algorithm

Fairmont State University
Advised by: Dr. Joseph Riesen

The RSA Algorithm is the method of encryption used in public-key cryptography. This "unbreakable" form of encryption was a breakthrough in the late 70's. The presentation given will cover how the algorithm works and why it works, followed by a proof of why it works as well as a small example.

11:10-11:25

Lisa Curll<br>To Save a Species<br>Youngstown State University<br>Advised by: Dr. Nathan P. Ritchey

Room 1062

Populations across the planet are at risk of extinction. Ecologists have been working to save them for generations, but numbers are still in decline. How can we see when a species is at risk, and to what degree is it endangered? Surprisingly, these questions can be answered by relatively simple usage of population projection matrices and eigenvalues. This simple strategy is unfortunately not recognized by many biologists, and I aim to make this process more understandable for everyone. During this talk we will examine and use the method based on a real population of spotted owls. The only question left will be, "What next?"

## 11:10-11:25

## John Hoffman

Room 2057

## Solution to PME Problem 1189

Youngstown State University
Advised by: Dr. Jacek Fabrykowski

We will explore a solution to PME Problem 1189. Problem 1189 states, "If $F_{n}$ denotes the $n$th Fibonacci number, show that $F_{4 n+2}-(2 n+1)$ is divisible by 5 ." We will explore a proof of this fact and offer another problem that can be solved using similar techniques.

Derivation of the Modern Equation of an Ellipse<br>Kent State University<br>Advised by: Dr. Morley Davidson

In ancient times, the conics were defined as the shapes created by intersecting a cone with a plane in various positions. But how did mathematicians arrive at the modern equations we use today? Specifically, I will show that a rectangular coordinate system can be chosen such that a locus of points on a given cone can be represented as the equation of an ellipse in standard form.

11:30-11:45
Sarah Ritchey
Room 1111

# Selecting a Spouse: It's All in the Mathematics 

Kennedy Catholic HS
Advised by: Dr. Nathan P. Ritchey

The marriage problem, which is also known as the secretary problem, is a famous problem in mathematics. Under the assumptions of the problem a fixed number of possible spouses can be "interviewed", one at a time, and in random order. At some point in the process, the person being interviewed is chosen. Also, for this problem, it is not possible to go back and select a person that was previously interviewed and passed over.

It turns out that the optimal solution strategy is quite interesting, involving probability, series, limits, and the famous number $e$. Other assumptions and the optimal solution strategy will be presented so that attendees will be able to choose the best spouse.

Room 1120

# Partial Differential Equation Model of Traffic Flow 

Youngstown State University
Advised by: Dr. George Yates

This talk will discuss the use of partial differential equations and the method of characteristics to model a traffic flow. The car density profile will be explored as a function of time and crash avoidance discussed. The behavior and accuracy of the model will be examined along the characteristic curves.

11:30-11:45
Christian Woods
Room 1062

Modeling Self-Sacrificing Bacteria<br>University of Pittsburgh<br>Advised by: Dr. Jonathan Rubin

The innate immune system relies heavily on the inflammatory response when destructive bacteria find their way into the body. However, biologists have discovered that in some cases inflammation can be more beneficial than harmful to a group of invading bacteria. Exactly why this is so is yet to be proven, but researchers have suggested two prominent theories known as the differential killing hypothesis and the food hypothesis. My research group has attempted to model these two possible explanations using logistic models as a basis for our increasingly complex systems of differential equations.

I will discuss the derivation of these models, as well as their fixed points. I will then use visuals produced by the differential equations program XPP and Matlab to examine the bifurcations of the fixed points, and as a result the conditions necessary for successful invasion by the pathogens. The talk will end with a brief description of the improvements our group has made to the initial models and how we plan to adapt our methods to work with these changes.

Kent State University
Advised by: Dr. Stephen Gagola

Virtual worlds such as Second Life are an emerging technology that can be used in an educational setting. 3D virtual worlds can be used to explain concepts of mathematics in an enjoyable visual manner to encourage creativity and motivate original thought. This talk is about research into developing educational lessons in mathematics using various virtual world technologies.

Kent State University

Advised by: Dr. Laura Smithies

I will give examples and prove the logical equivalence of epsilon-delta, topological, and sequential definitions of point continuity over the real numbers.

Jeana Stevens
Room 1111

# Maximal Circle Radius in Hypercubes 

Youngstown State University
Advised by: Dr. Thomas Smotzer

In this presentation we will determine the largest possible radius of a circle contained within a 3dimensional hypercube. We will then extend our result to higher dimensional cubes. This work is based on problem B3 from the most recent Putnam exam.

11:50-12:05<br>Neil Sandberg<br>An Alternative Approach to Compression Technique Evaluation<br>Case Western Reserve University<br>Advised by: Dr. David Singer

Room 1120

Conventionally, the effectiveness of data compression algorithms has been measured by running the algorithms with a common set of files as input. The set, currently known as the Canterbury Corpus, consists of a handful of small, representative files of various types. Since testing the algorithm against all possible files is not only impossible but also useless, the approach was designed to quickly gain data regarding how well a given algorithm can be expected to perform on a typical file. While the approach is valid, it comes with a few weaknesses; thus, in this paper an alternative strategy is presented. In accordance with the recent Web 2.0 trend, this paper proposes the existence of an application that allows data contributors to upload files to a host system which compresses the files, stores metrics on the compression, and exposes the database of results via data tables and charts. Elementary concepts will be introduced, followed by brief introductions to the algorithms to be compared. Then, design motives for the system will be presented, followed by a demonstration of the system actually developed by the author. Finally, results obtained with the application will be presented, along with analysis that hopes to explain the results. It is the hope of the author that, in response to the work done in this project, a production-level adaptation of this idea be created and maintained by a person or group with more resources for doing so.

In Flatland, females are lines, and males are $n$-sided regular polygons. When they produce a new male it has $n+1$ sides. Let's investigate some of the geometric aspects of Flatland procreation.

Jeff Burdette The Stochastic Group

Kent State University
Advised by: Dr. Mikhail Chebotar and Dr. Artem Zvavitch

Room 1111 Advised by: Dr. Mikhail Chebotar and Dr. Artem Zvavitch

I will briefly discuss Markov Chains, as they are the motivation for stochastic matrices (matrices whose columns sum to 1). I will then prove that the set of all nonsingular stochastic $n \times n$ matrices over a field $F$ forms a group under matrix multiplication.

Andrew Hosie<br>The Calculus of Landing an Airplane<br>Edinboro University of PA<br>Advised by: Dr. Emily H. Sprague

We provide and explain the solution to the Applied Project "Where Should a Pilot Start Descent?" from Stewart's Calculus text and provide some remarks to relate our result to the "Miracle on the Hudson".

1:00-1:15
Vikram Raghu
Room 1062

A Novel Mathematical Model of Human Lactation<br>University of Pittsburgh<br>Advised by: Dr. Jonathan Rubin

Lactation in human mothers is controlled by the interactions of several factors. Chemically, the major hormonal factors involved are prolactin, which stimulates milk component synthesis, and oxytocin, which upon suckling is released to stimulate milk ejection. The details of prolactin control of milk synthesis during established lactation remain a mystery. This work presents a novel mathematical model of the cascade of interactions governing milk synthesis and ejection during established lactation. The model was used to study alternative hypotheses concerning the effects of suckling on prolactin receptor regulation.

Pierre de Fermat stated a problem which was partially solved by William Brouncker and whose solution was completed by Leonhard Euler. This problem in number theory has applications to approximating square roots of primes using hyperbolas. We will discuss which primes work the best for this method.

Thermal Interrogation of Porous Materials<br>Youngstown State University<br>Advised by: Dr. H.T. Banks

Many modern aero and space structures are composed of composite materials containing significant porosity. Although nondestructive analysis techniques have been developed to detect damage in homogeneous materials, little research has been done on heterogeneous materials. We first use different probability distributions to randomly generate pores in a compartment to create our heterogeneous material. We then use the heat equation to simulate flash heating of the compartment along one of its boundaries. Temperature data along the boundaries is recorded and then analyzed to distinguish differences between the undamaged and damaged materials. This talk is based on and REU at North Carolina State University.

Room 1120

The Cauchy Condensation Test is an extremely interesting test used to determine the convergence and divergence of infinite series. We will explore a proof and examples using the test as well as examine the expansion for the general theorem.

1:20-1:35
Christopher Horvat
Room 1062
A Mathematical Model of Necrotizing Enterocolitis
University of Pittsburgh
Advised by: Dr. Ivan Yotov

Necrotizing Enterocolitis (NEC) is a prevalent neonatal disease considered the greatest threat to the survival of preterm infants. We present a mathematical model that reproduces the body's inflammatory response to damage of the tissue layers surrounding blood vessels. Using this model, we attempt to examine the effects of various treatment methods for NEC, most notably the concept that breastfeeding drastically reduces the mortality rate.

Let us take you on a journey through the first three dimensions (and beyond!) as we recall our epic battle with the derivation of a formula for the $n$-dimensional volume of a Hypersphere.

1:40-1:55<br>Chim Chune Ko \& Aron Siegel<br>Real Numbers Under Alternative Arithmetic<br>Clarion University<br>Advised by: Dr. Jon Beal

In this talk we will explore various properties that arise when the definition of multiplication and addition are changed for the real numbers. We define

$$
\begin{gathered}
a \oplus b=a+b-1 \\
\text { and } \\
a \odot b=a+b-a b .
\end{gathered}
$$

We will explore identities and inverses, exponents, and determine how to define subtraction and division.

Dylan Sorge
The Margulis Napkin Problem
Kent State University
Advised by: Dr. Stephen Gagola

The napkin folding problem in geometry explores whether folding a square or a rectangular napkin can increase its perimeter. This presentation will examine the mathematics of one solution to the problem as it was solved by Robert J. Lang.

Thomas Reid
Analysis of topographic maps from fMRI data
Case Western Reserve University
Advised by: Dr. Anthony I. Jack

Room 1062

In recent years, a number of studies have attempted to demonstrate the presence of retinotopic maps in the occipital, parietal and frontal cortex. The evidence for these maps has come from inspection of color-coded maps based on phase-encoded visual field stimulation. However, this method is qualitative in nature and the maps have not been consistent between groups and subjects, thus leaving the important question of what these mappings look like or whether they even exist unanswered. Our research involves regression analysis of BOLD fMRI data in an attempt to establish a more quantitatively rigorous assessment of the presence of topographical maps on the cortical surface. If time permits, we would like to extend this mapping from the occipital lobe to higher cortical areas, and examine the faithfulness of our mapping for when considering both attended and unattended stimuli.

Richard Ligo \& John-Tyler Soltys<br>Magic via Modular Arithmetic<br>Westminster College<br>Advised by: Dr. Carolyn Cuff

We examine a classic card trick where the magician is able to read the participant's mind to determine which card was secretly chosen. Varying two parameters of the problem, we note that the card trick will fail under some conditions. The data collected is presented. We establish equations to specify failure: $x$ $\bmod 2 p$ if $p=2 i+1$ and $x \bmod (p+1)$ if $p=2 i$, where $x$ equals the number of cards and $p$ equals the number of piles.

2:00-2:15

Emily Cunningham \& Josh Fitzgerald When Positive Numbers Aren't<br>Clarion University<br>Advised by: Dr. Jon Beal

Room 1111

In a continuation of the "circle math" talk, we will determine what positive and negative mean with respect to the operations

$$
a \oplus b=a+b-1
$$

and

$$
a \odot b=a+b-a b .
$$

In order to do this, we first must determine what it means to be a "positive" number. This discussion will also cover the definitions of powers and roots of a number. That is, for $n$, a positive integer, what exactly is $x$ raised to the $n$th power $\left(x^{\circ n}\right)$ and the $n$th root of $x(\sqrt[\circ n]{x})$.

2:00-2:15

## Andrew Johnson

Room 1120
The Great American Road Trip
Mount Union College
Advised by: Dr. Sherri Brugh

Have you ever been on a road trip? How many states did you drive through? If you wanted to drive through all forty-eight continental states, what would be the shortest route? The answer is the solution to The Great American Road trip problem.

2:00-2:15

# Kristen Pueschel <br> Tracing the Motion of Migrating Cells by Image Analysis 

Room 1062
University of Pittsburgh
Advised by: Dr. David Swigon

Imaging is often used in cell biology because it allows for gathering in vivo data without the disassembly and disruption of biological systems. Image analysis is then applied to the images to extract data useful for model-calibration. I will talk about two image-analysis projects related to migration of different types of cells. In the first project, traces of cell motion are gathered from movies of migrating enterocytes, i.e., cells that form the lining of intestines. The data will be used to calibrate a two-dimensional continuum PDE model of wound-healing in a gut. For the second project we analyze the differences in motion of stem cells versus differentiated cells, based on movies of cells moving along micro-grooves.

The Buffon needle problem was proposed in 1733 by French mathematician Georges-Louis Leclerc, Comte de Buffon. I am going to state the problem and several important results from probability theory. I will then provide the solution and explain how this process lead to an early method for estimating the value of $p i$.

# An Intriguing Limit with Euler in Mind 

Youngstown State University
Advised by: Dr. Jacek Fabrykowski

For my senior project I am considering the limit,

$$
\lim _{n \rightarrow \infty} \sqrt[n]{\operatorname{lcm} 1,2, \ldots, n}
$$

During this presentation, I will present the tools necessary to evaluate it including arithmetic functions, summation by parts (Abel Summation) and other topics from number theory. An outline of the solution will be given using these tools.

2:20-2:35
Michelle Cordier
Room 1120
Special Colorings of Graphs
Kent State University
Advised by: Dr. Stephen Gagola

This talk will consider the number of ways to special color a graph's vertices. Looking at special graphs (complete, path, and cycle graphs) I will find formulas for colorings. I will then conclude with an example.

# 2009 MCM / ICM - COMAP Modeling Problems 

Continuous Modeling (Problem A)

Many cities and communities have traffic circles-from large ones with many lanes in the circle (such as at the Arc de Triomphe in Paris and the Victory Monument in Bangkok) to small ones with one or two lanes in the circle. Some of these traffic circles position a stop sign or a yield sign on every incoming road that gives priority to traffic already in the circle; some position a yield sign in the circle at each incoming road to give priority to incoming traffic; and some position a traffic light on each incoming road (with no right turn allowed on a red light). Other designs may also be possible.

The goal of this problem is to use a model to determine how best to control traffic flow in, around, and out of a circle. State clearly the objective(s) you use in your model for making the optimal choice as well as the factors that affect this choice. Include a Technical Summary of not more than two double-spaced pages that explains to a Traffic Engineer how to use your model to help choose the appropriate flowcontrol method for any specific traffic circle. That is, summarize the conditions under which each type of traffic-control method should be used. When traffic lights are recommended, explain a method for determining how many seconds each light should remain green (which may vary according to the time of day and other factors). Illustrate how your model works with specific examples.

## Discrete Modeling (Problem B)

This question involves the "energy" consequences of the cell phone revolution. Cell phone usage is mushrooming, and many people are using cell phones and giving up their landline telephones. What is the consequence of this in terms of electricity use? Every cell phone comes with a battery and a recharger.

Requirement 1: Consider the current US, a country of about 300 million people. Estimate from available data the number H of households, with m members each, that in the past were serviced by landlines. Now, suppose that all the landlines are replaced by cell phones; that is, each of the members of the household has a cell phone. Model the consequences of this change for electricity utilization in the current US, both during the transition and during the steady state. The analysis should take into account the need for charging the batteries of the cell phones, as well as the fact that cell phones do not last as long as landline phones (for example, the cell phones get lost and break).

Requirement 2: Consider a second "Pseudo US"-a country of about 300 million people with about the same economic status as the current US. However, this emerging country has neither landlines nor cell phones. What is the optimal way of providing phone service to this country from an energy perspective? Of course, cell phones have many social consequences and uses that landline phones do not allow. A discussion of the broad and hidden consequences of having only landlines, only cell phones, or a mixture of the two is welcomed.

Requirement 3: Cell phones periodically need to be recharged. However, many people always keep their recharger plugged in. Additionally, many people charge their phones every night, whether they need to be recharged or not. Model the energy costs of this wasteful practice for a Pseudo US based upon your answer to Requirement 2. Assume that the Pseudo US supplies electricity from oil. Interpret your results in terms of barrels of oil.

Requirement 4: Estimates vary on the amount of energy that is used by various recharger types (TV, DVR, computer peripherals, and so forth) when left plugged in but not charging the device. Use accurate data to model the energy wasted by the current US in terms of barrels of oil per day.

Requirement 5: Now consider population and economic growth over the next 50 years. How might a typical Pseudo US grow? For each 10 years for the next 50 years, predict the energy needs for providing phone service based upon your analysis in the first three requirements. Again, assume electricity is provided from oil. Interpret your predictions in term of barrels of oil.

## 2009 PME National Meeting at MAA MathFest

Please join us at this year's meeting to be held August 6 through August 9, 2009, in Portland, Oregon. Students are invited to give fifteen minute talks on any mathematical topic or application in areas such as statistics, computing, or operations research. Topics including expository research, interesting applications, problems, etc. are also welcome. Transportation reimbursement is also available to those who qualify. Visit the National Pi Mu Epsilon website at http://www.math-pme.org for more details.

## A Warm Welcome to the Participating Schools:

- Case Western Reserve University
- Clarion University of Pennsylvania
- Cleveland State University
- Edinboro University of Pennsylvania
- Fairmont State University
- John Carroll University
- Kennedy Catholic High School
- Kent State University
- Lakeland Community College
- Mount Union College
- Slippery Rock University
- University of Pittsburgh
- West Liberty State College
- Westminster College
- Youngstown State University


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Special thanks to the Department of Mathematics and Statistics and the Center for Undergraduate Research in Mathematics (CURMath) at Youngstown State University.

